## UNCERTAINTY, INSURANCE, AND RISKY ASSETS

## Problem \#1

There are five participants in a THE ONLY CHANCE ${ }^{\text {TM }}$ show. We have the following information about them:

| UTILITY FUNCTION | INITIAL WEALTH | APPROACH TO RISK |
| :---: | :---: | :---: |
| Mr. A $-U(W)=W^{0.5}$ | 1000 | ......................... |
| Mr. B $-U(W)=\ln (W)$ | 2000 | . $\cdot$..................... |
| Mr. C $-U(W)=W^{2}$ | 100 | ....................... |
| Mr. D - U $W$ ( $)=W$ | no data | ........................ |
| Mr. E - a rational person | no data | no data |

a) Which of the players is a risk-lover and which is risk-averse?
b) Game \#1: Mr. A and Mr. C are faced with the following problem. They are offered participation in the show, where they are always allowed to chose between two gates. One of them hides the price of 5000, while the other has nothing to offer. Obviously they do not know which is which. If somebody decides not to play, he receives compensation of 500 . Which of the players will prefer to play and which will settle for the compensation? Provide a graphical and algebraic solution.
c) Game \#2: Mr. E was confronted with the following choice.


He chose the first lottery. In the second round of the show he may choose from the following options:


If he is rational, which of the lotteries will he choose in the second round?
d) Game \#3: Mr. B has already won 1000. In the second round he chooses among three gates. Behind two of them there is 5000 waiting for him. If he chooses one of them, he keeps his 1000 and gets this additional 5000 . However, if he chooses the third gate, not only does he win nothing, but he also has to return his 1000 won earlier in the show. Will he participate in this game?

## Problem \#2

Andrew located his savings in an internet company. If he is right about the predictions of economic growth and network development, in a year he should have $\$ 81$. However, if he is not right, his shares will only be worth $\$ 9$. He estimates that he can be wrong and right with the same probability. His utility function is a logarithm of wealth with the base of 3 . For how much would he be willing to sell his shares?

## Problem \#3

Criminality has recently become a serious problem in Poland. The main motive for committing crimes has been an economic one. Below you will find a way to analyze this phenomenon with reference to the theory of choice under uncertainty. For a potential criminal the expected income is not the only factor that counts - the potential penalty and risk are also important. Therefore, committing a crime can be represented by the game presented below. Assume that the criminal is risk-neutral.
a) Simplify the game presented below.
b) What is more harmful for the criminal: increasing the crime detection rate or the probability that conviction will be unavoidable? (Hint: Specify how a change of the relevant probabilities ( $p, q$ ) influences the criminal's utility)
c) The probabilities of detecting a crime and conviction are currently identical: $q=p=0.5$. Determine the guaranteed income (which does not originate from the crime) that will cause the criminal to undertake legal work.
d) The problem of increasing the severity of the penalty is often discussed. On the basis of the game given above determine what is more harmful for the criminal: imposing a more severe penalty or increasing the likeliness of conviction?

where:
$p$ - probability of being caught
$q$ - probability of being convicted

## Problem \#4

Natasha's utility function is given by the formula $U(Y)=Y^{1 / 2}$, where $Y$ stands for her annual income in thousand USD. She currently earns 10000 USD $(Y=10)$ and can also (for sure) earn this amount next year. However, she was offered a new job with a $50 \%$ chance for a salary of 16000 USD and 5000 USD otherwise.
a) Will she accept this job offer?
b) Assume she undertakes the new job. Would she be willing to purchase insurance against the changes in income at the new job? If yes, how much would she be willing to pay for it?

## Problem \#5

There exists one company in the automobile insurance market. A typical person insured by this company owns a car worth 30000 zloty and is risk-neutral. The probability of a car being stolen during a certain year is $1 / 1000$. The company offers partial insurance (pays back $2 / 3$ of the car's value). The insurance premium amounts to 30 zloty. Another firm enters the market. The insurance offered by this company is more expensive and costs 35 zloty. However it also includes satellite monitoring of cars. This company also offers partial insurance (pays back $2 / 3$ of the car's value). Calculate by how much would the probability of the car being stolen have to decrease for a person buying car insurance to choose the second company (insurance is mandatory).

## Problem \#6

Assume that somebody invested 1000000 USD in a project. The perspectives were to earn 600000 USD after the first year and the same amount a year later. With the interest rate of $8 \%$ this would imply a Net Present Value of approximately 70000 USD. How would NPV change in there was a delay of one year?

## Problem \#7

An investor has two business opportunities to choose from. Project A gives him 10000 zloty or 3000 zloty with the probability of $1 / 2$, respectively. If he chooses project $B$, however, his payoffs will be 0 with the probability of $1 / 9,2000$ zloty with the probability of 7/9 or 40000 zloty with the probability of $1 / 9$.
a) What is the expected value and the variance of these two investment opportunities?
b) Are we able to unambiguously determine the investor's choice on the basis of the calculations from point a)?
c) If his utility function takes the form $U(W)=W^{1 / 2}$, where $W$ stands for wealth, what will he choose?

