

Problem #1

The utility functions, which you encountered in problems solved in class, represent certain “model” preferences. Of course, we can deal with other types of preferences and, accordingly, utility functions may take different forms. Below you will find utility functions, which you did not see in class and which you will probably rarely encounter during your further studies. Nevertheless, it is worth taking a closer look at them. Graph the hypothetical map of indifference curves for these functions and find the marginal utilities MU_1 and MU_2 , as well as MRS. Are marginal utilities for these functions positive and does the law of diminishing marginal rate of substitution operate? Do they fulfill the rational choice axioms? What kind of preferences are represented by these functions? Can they reflect real preferences?

- a) $U = 3X_1^2 + 5X_2$
- b) $U = 20 - (X_1 - 10)^2 - (X_2 - 10)^2$
- c) $U = (X_1 - X_2)^2$

Problem #2

In kindergarten little kids like to trade. Each of them has building blocks and sweets. You know that 10 kids always want to have as many blocks as sweets. At the end of today’s playing time each kid in this group has 10 sweets and 20 blocks. For 6 kids building blocks and sweets are perfect substitutes. Since they neither play with the blocks, nor eat sweets, they treat them as objects, which they should have. Each of the kids in this group has 20 sweets and 10 blocks. The final group of 6 kids has preferences typical for sweet-lovers, which may be described by function $U = B \cdot S^2$ (where B stands for building blocks and S – for sweets). Each of the kids in this group has 4 sweets and 30 blocks. The kindergarten teacher decided to “pacify” little constructors and proposed that for every building block she will give them 4 sweets. How many blocks will she be able to collect?

Problem #3

In two competing supermarkets (A and B) shampoos and conditioners are on sale. The initial (pre-sale) price of the shampoo is 20 *zloty*, while the conditioner costs 15 *zloty*.

- Supermarket A will sell packages, i.e. if we buy a shampoo and a conditioner together we will pay 25 *zloty* (these products cannot be bought separately anymore).
- Supermarket B will be selling both products independently. Buying two bottles of any of them will give you the third one for free (works both for shampoos and conditioners but cannot be mixed, i.e. if we buy two shampoos and two conditioners we will get a free shampoo and a free conditioner).

You are supposed to do shopping for your family and you can spend no more than 100 *zloty* on shampoos and conditioners. Where will you go shopping if you know that in your family one conditioner is used for every three shampoos? The utility function of your family can be approximated by $U = \min\{\text{shampoo}, 3 \cdot \text{conditioner}\}$.

(Simplification: Assume the budget constraint for countable goods)