REVEALED PREFERENCES AND INDICES

Problem #1 (illustration)
Below you see a typical text describing macroeconomic outlooks. All variables are intuitive and should not require detailed explanations. CPI is a measure of price changes, i.e. inflation.
"After the recent drop in the value of the Polish złoty the market was calmed by the macro reports published by the Main Statistical Office on Thursday. According to these results prices grew by 3.4% which is less than 3.7% (as expected). At the same time, sales grew by 4.1%, whereas growth of only 3.7% was expected."
Can we infer from this text whether the society as a whole is better or worse off? If we knew the society's utility function, it would be easy to answer this question. Should the consumer be on a higher utility curve after the change, his situation has improved and the problem is solved. The problem below is an illustration of this approach.
For simplicity assume only two types of goods are consumed in the economy, namely food (*F*) and clothes (*C*). The food prices fell from 2 to 1.5 *zloty* per unit, while prices of clothes increased from 3 to 3.5 *zloty* per unit. For some consumers income has increased while for others it remained unchanged. On the basis of the table below analyze the situation of four representative consumers. Their utility functions are given in the table.

Consumer	Utility function	Disposable income before changes	Disposable income after changes
Mrs. A	$U = min\ \{F,3C\}$	1000	1200
Mr. B	$U = F^{0.4}C^{0.6}$	1500	1500
Mr. C	$U = F + C$	2000	2500
Mrs. D	$U = F^{0.2}C^{0.8}$	6000	5500

In this problem one should compare the utility values for the optimal consumption before-change-baskets and after-change-baskets. It is handy to use the formulae for demand, depending on the price and income, for these utility functions.

Problem #2
The households of pensioners usually spend about 60% of their income on food. Assume that food prices increase by 20%. By how much do we need to raise pensions to guarantee that the economic situation of an average household of this social group does not deteriorate? For what change of pensions are we unable to predict (on the basis of the given data) the direction of changes in the living standards? What other data would we need to have to be able to predict the latter changes?

Problem #3
The Private Entrepreneurs Association estimated that on average a typical employee spends 50% of his income on food. Assume food prices increase by 20%. Furthermore, assume that the Association aims at increasing the income of the employees in order to maintain their utility at a constant level (thereby guaranteeing the same effort level).
a) Assume we do not know the employees' utility functions. By how much (in %) should private entrepreneurs increase the nominal income (wage) of their employees in order to be sure that they achieve their goals? Provide a reasoning for your answer.
b) Assume that the employees' utility function is given by the formula $u(f,i) = f^{0.5}i^{0.5}$, where *f* denotes food and *i* all other goods. By how much (in %) should private entrepreneurs increase the nominal income (wage) of their employees in order to be sure that they now achieve their goals? What is the share of income spent on food after price increase?
c) Can you think of a situation where the entrepreneurs would not be forced to pay anything but, nevertheless, would achieve their goal?

Problem #4
Kate and Tom are a couple. They differ pretty much in everything, even as regards the perception of economic changes. For Kate, books and opera are the only things that count. She perceives her economic situation *via* the ability to buy these two goods. She has recently compared her average monthly expenses in 2003 (last year of studies) and now (2005).

Year	Number of books bought (x_1)	Number of concerts (x_2)	Average book price (p_1)	Average concert price (p_2)
2003	10	8	12	15
2005	12	6	15	20

If we compared expenses, Kate spent much more in 2005 for books and tickets than she did in 2003. We could say she spent 25% more in 2005 than in 2003. The expenses index amounts to 1.25 (=300/240).

Year	x_1*p_1	x_2*p_2	Sum of expenses
2003	120	120	240
2005	180	120	300

Does this mean that Kate's situation has improved? Not really. She spent more but the prices of both books and tickets have increased. Therefore, the increase in expenses follows from changes both in consumption patterns and in prices. The answer to the question of "what happened?" needs to be decomposed into two parts: What happened with prices and how did real income change?

How do we measure the change in real income? Kate perceives her situation on the basis of the composition of the basket containing the two goods. Hence, the answer about the real income comes down to verifying whether she would spend more on her goods if prices remained unchanged. The only question remains which prices should we take into account? This is insignificant – we can take either the 2005 or the 2003 prices.

Number of books bought (x_1)	Number of concerts (x_2)	Average book price (p_1) /2003 prices/	Average concert price (p_2) /2003 prices/	x_1*p_1	x_2*p_2	Sum of expenses
10	8	12	15	120	120	240
12	6	12	15	144	90	234

On the basis of the 2003 prices Kate spent 2.5% less in 2005 than in 2003 (234/240 = 0.975). Her real income measured this way decreased. We will obtain a similar result on the basis of the 2005 prices.

Number of books bought (x_1)	Number of concerts (x_2)	Average book price (p_1) /2005 prices/	Average concert price (p_2) /2005 prices/	x_1*p_1	x_2*p_2	Sum of expenses
10	8	15	20	150	160	310
12	6	15	20	180	120	300

The result is even worse. On the basis of the 2005 prices Kate spent 3.3% less in 2005 than in 2003 (300/310 = 0.9677).
It is already noticeable that the increase in expenses is driven by prices. How do we measure them? The idea is similar as above. We calculate the weighted average where the 2003 or 2005 bundles are used as weights.

Number of books bought (x_1) /2003 quantities/	Number of concerts (x_2) /2003 quantities/	Average book price (p_1)	Average concert price (p_2)	x_1*p_1	x_2*p_2	Sum of expenses
10	8	12	15	120	120	240
10	8	15	20	150	160	310

Number of books bought (x_1) /2005 quantities/	Number of concerts (x_2) /2005 quantities/	Average book price (p_1)	Average concert price (p_2)	x_1*p_1	x_2*p_2	Sum of expenses
12	6	12	15	144	90	234
12	6	15	20	180	120	300

Prices increased by over 29% if we reason on the basis of the 2003 bundle (310/240 = 1.29). We obtain a similar value (28%) for the 2005 bundle.

a) In the first part of the problem (above) we calculated quantity and price indices. Provide the proper names of these indices (Paasche and Laspeyers indices).
b) Tom is a PC maniac. He consumes CDs and hard drive memory. Why is Tom perceiving his situation completely differently than Kate? Calculate the price and quantity indices on the basis of the data given below.

Year	CDs	HD memory in GBs	Average CD price	Average HD memory price (per GB)
2003	400	40	1.2	35
2005	370	250	0.7	20

c) Kate and Tom constitute one household for the Main Statistical Office. How would the Office assess the situation of this household? Create a joint basket for Kate and Tom. Calculate the price and quantity indices. (Note that Kate calculated her monthly expenses, while Tom calculated his yearly expenses.)

Problem #5
A government agency for strategic studies decided to query the situation of students originating from small villages. They chose two months for comparison – October 2003 and October 2004. The price of a room in a students' residence increased from 300 to 350 *zloty*, the price of a student restaurant card – from 400 to 450 *zloty*, the price of the communal transportation ticket – from 30 to 36 *zloty*, the price of an average book – from 25 to 30 *zloty*, the price of a cinema ticket – from 14 to 18 *zloty*. The prices of other goods remained unchanged. We only have data for 2004 as regards the consumption basket for a typical student. It appears that monthly each of them pays for their room, the student restaurant card, the transportation ticket, 2 books and 6 cinema tickets. The remainder is spent on goods for which prices remained unchanged. The expenses of an average student increased from 1000 to 1100 *zloty*. Has his/her situation improved or worsened between 2003 and 2004?