MICROECONOMICS I
2010/2011

EXAMPLES OF FINAL TEST PROBLEMS

(DIRECT TRANSLATION OF THE MATERIAL PROVIDED IN POLISH BY DR. MAGDALENA KANIEWSKA)

Remarks (KMSz):
1) points for specific parts of the solution are given in brackets
2) some graphs may be not to scale
3) some solutions provided by Dr. Kaniewska apply methods we rarely used in class (e.g. Lagrange auxiliary function), usually it is possible to solve them using other approaches that will be more familiar to you too

SET 1

Problem #1

You are considering whether to buy a cottage house and what you are interested in is the distance of the house from the forest (currently, living in a city, you have about 55 km to the forest). Obvoiusly, the closer to the forest, the higher the price of the house (i.e. the further from the forest, the lower the price) according to the following formula: *price of the house* = 30 000 – 400*d*, where *d* is the distance from the forest in km. Your entire monetary income amounts to M = 500 000. You do not like spending money but you enjoy spending time in the forest. Your utility relating to money held in cash and proximity of the forest may be expressed by the following formula: $U(m,d) = M - 4d^3$. Are you going to buy the house? If so, what kind of a house are you going to buy? Remember that the amount of money you hold in cash decreases by the price of the house when you buy the latter.

Solution:

$U(M,d) = M - 4d^3 - (30\,000 - 400d)$ (**2 p.**) */KMSz comment: Lagrange auxiliary function/*

$$U' = \frac{\partial U}{\partial d} = -12d^2 + 400 = 0$$

$$d = \pm\frac{10}{\sqrt{3}}$$

(first-order condition for function U max/min: **1 p. + 1 p. + 1 p.**)

Maximum of U is for $d = \frac{10}{\sqrt{3}}$ /formal proof: $U'' = -24d$ so $U''(d = \frac{10}{\sqrt{3}}) < 0$ which is then the maximum and $U''(d = -\frac{10}{\sqrt{3}}) > 0$ which is the minimum./

Problem #2
In our lives we face many constraints. So does Donna in the problem below, where she faces the financial and time constraint. She has been unemployed for the last 6 months and feels that she should find a new job. Her dad suggests that she should visit some shops in the second part of the town where she lives (she lives in a residential district, where there are no shops) in person to ask around for a job. Each such visit in person costs Donna 5 zloty for transportation and dry-cleaning (Donna believes that she can only impress the potential employer when wearing clean clothes) and requires devoting 5 hours for traveling there and back. Donna's mom suggests that visiting potential employers in person is a waste of time and that Donna should instead send her applications by post, which costs just 1 zloty and requires 30 minutes time for sending each application. Donna can weekly devote 30 hours to searching for a job and can spend a maximum of 50 zloty per week.

What does Donna's constraint with relation to different possibilities of searching for the job look like? Remember that there are 2 constraints.

Solution:

Application in person: 5 zloty and 5 hours
Application by post: 1 zloty and 0.5 hours
Financial constraint: 50 zloty
Time constraint: 30 hours

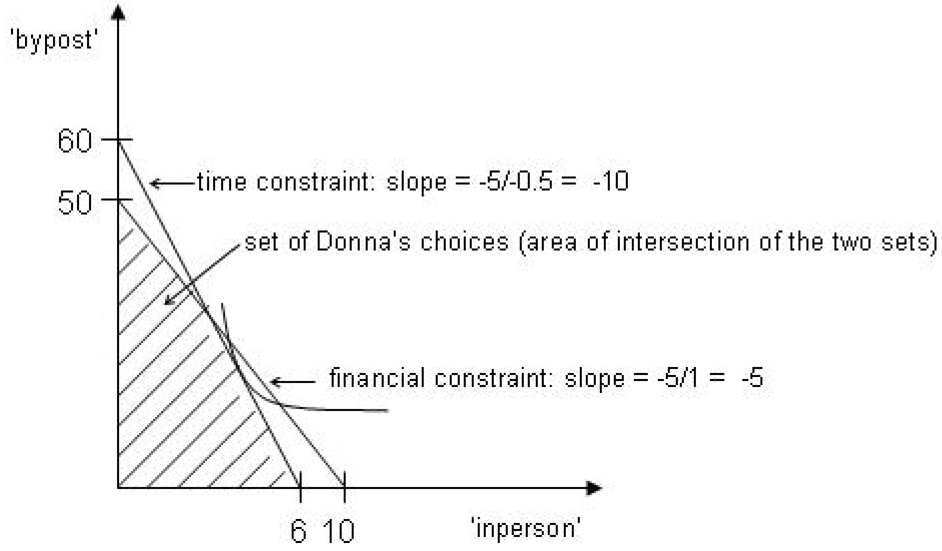
Financial constraint: 5 * 'inperson' + 1 * 'bypost' = 50 (**1 p.**)
Time constraint: 5 * 'inperson' + 0.5 * 'bypost' = 30 (**1 p.**)

Graph the set representing Donna's possible choices.

Solution:

5 * 'inperson' + 1 * 'bypost' = 50
and
5 * 'inperson' + 0.5 * 'bypost' = 30

'inperson' = 2 and 'bypost' = 40 (**1 p.**)
(the intersection of both constraints)



(**2 p. + 2 p.**)

What is Donna's choice going to be, i.e. which method of searching for a job will she choose, if her preferences reagrding such methods are given by the following function: *U*('inperson','bypost') = 'inperson' * 'bypost' (Cobb-Douglas function)? Find the algebraic solution and include it on the graph above. Remember that the line with a kink is described by two formulae.

Solution:

Since the constraint has a kink, we must apply maximization within two intervals:

1. 'inperson' $\in < 0,2 >$, 'bypost' $\in < 40,50 >$

L = 'inperson' * 'bypost' – λ(5 * 'inperson' + 1 * 'bypost' - 50)

$$\frac{\partial L}{\partial' inperson'} = 'bypost' - 5\lambda = 0$$

$$\frac{\partial L}{\partial' bypost'} = 'inperson' - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 5'inperson' + 'bypost' - 50 = 0$$

Finally:
'inperson' = 5 (**4 p.**)
'bypost' = 25
(contradiction, values not within the intervals)

2. 'inperson' $\in < 2,6 >$, 'bypost' $\in < 0,40 >$

L = 'inperson' * 'bypost' – λ(5 * 'inperson' + 0.5 * 'bypost' - 30)

$$\frac{\partial L}{\partial' inperson'} =' bypost' - 5\lambda = 0$$

$$\frac{\partial L}{\partial' bypost'} =' inperson' - 0.5\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 5' inperson' + 0.5' bypost' - 30 = 0$$

Finally:
'inperson' = 3 (**4 p.**)
'bypost' = 30
This is our solution (falls within the intervals).

Problem #3
Dan always consumes strawberry and vanilla ice-cream in a constant proportion (what makes his friends laugh at him). For very two portions of strawberry ice-cream he always adds three portions of vanilla ice-cream. Of course, he is also interested in multiples of these numbers (in a constant proportion).

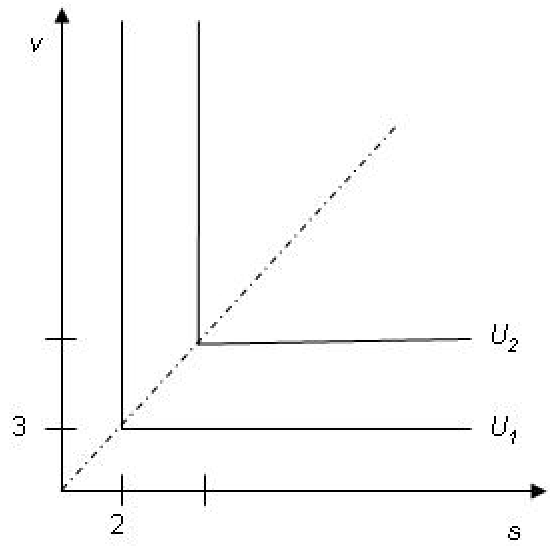
Provide the formula for the utility function for such preferences.

Solution:

$U(s,v) = \min\{3s, 2v\}$, where s – strawberry portions, v – vanilla portions (**2 p.**)

Graph such preferences.

Solution:



(**4 p.**)

Find MRS.

Solution:

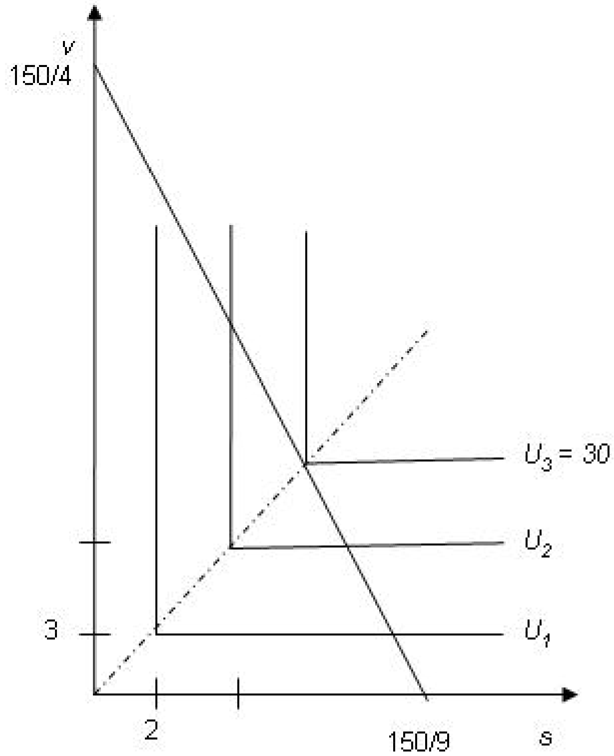
MRS is equal to 0 or infinity, MRS does not exist. (**2 p.**)

What is Dan's optimum choice if he spends 150 a month on ice-cream and a portion of strawberry ice-cream costs 9, while a portion of vanilla ice-cream costs (provide an algebraic and graphical solution).

Solution:

We know that Dan's optimum choice must lie on the line where $3s = 2v$ and budget line, i.e.:
$9s + 4v = 150$ and
$3s = 2v$
That gives:
$s^* = 10$ (**4 p.**)
$v^* = 15$

$U(10,15) = \min\{3*10, 2*15) = 30$



(4 p.)

SET 2

Problem #1

You would like to have a cottage house so you are considering to buy one. The area, where you would like to buy the house, is known for its beautiful swamps and moors. Although the proximity of wilderness is something you desire, it is inevitably connected with high numbers of certainly less-desired mosquitoes. A special firm approximated the relationship between the price of a house and average number of mosquito-bites per minute as: *price of the house* = 150 000 – 300*j*, where *j* is the average number of mosquito-bites per minute. Your wellbeing worsens together with the increase in the number of expected bites per minute, so that your satisfaction from money in cash and expected number of bites per minute is described by the relationship: $U(M,j) = M - 3j^2$. Are you going to buy the house? If so, what kind of a house are you going to buy, if your initial wealth is *M*? Remember that the amount of money you hold in cash decreases by the price of the house when you buy the latter.

Solution:

$U(j) = M - 3j^2 - (150\,000 - 300j) = M - 150\,000 - 3j^2 + 300j$ **(2 p.)**

$U' = \frac{\partial U}{\partial j} = -6j + 300 = 0$ **(1 p. + 1 p. + 1 p.)**

$j = 50$

/$U'' = -6 < 0$ so $j = 50$ is a maximum/

$U(j) = M - 150\,000 - 3j^2 + 300j$

for $j = 50$ $U(j) = M - 150\,000 + 7\,500$

Yes, you will buy the house. When the house is not bought *j* is unknown and may approach ∞.

Problem #2

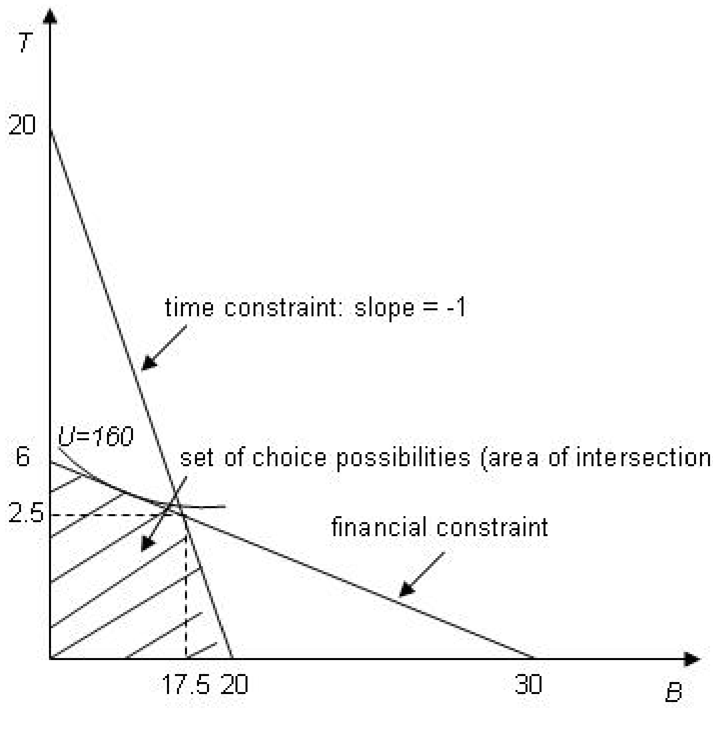
Warren spends his free time playing tennis and swimming. His utility function relating to these activities has the following form: $U = B * T^2$, where T – hours spent playing tennis, B – hours spent at the pool. An hour of swimming costs 10 zloty, while an hour of tennis – as much as 50 zloty. Warren has 20 hours at his disposal and a total of 300 zloty to devote to these two activities per month.

What does Warren's choice set relating to those different possibilities of spending free time look like? Provide a formal analysis and a graph. Remember Warren also has a time constraint.

Solution:

Financial constraint: $10B + 50T = 300$ (**1 p.**)
Time constraint: $B + T \leq 20$

$10B + 50T = 300$ and
$B + T = 20$
So: $T = 2.5$, $B = 17.5$ (**1 p.**)



(**2 p.**)

How many hours a month will he devote to tennis and how many – to pool? (calculate it and present on the graph above)

Solution:

$L = B * T^2 - \lambda(10B + 50T - 300)$
with $B \in <0,20>$ and $T \in <0,6>$

$$\frac{\partial L}{\partial B} = T^2 - 10\lambda = 0$$
$$\frac{\partial L}{\partial T} = 2TB - 50\lambda = 0 \qquad \textbf{(3 p.)}$$
$$\frac{\partial L}{\partial \lambda} = 10B + 50T - 300 = 0$$

So: $T^* = 4$ and $B^* = 10$

$U(10,4) = 10 * 4^2 = 160$

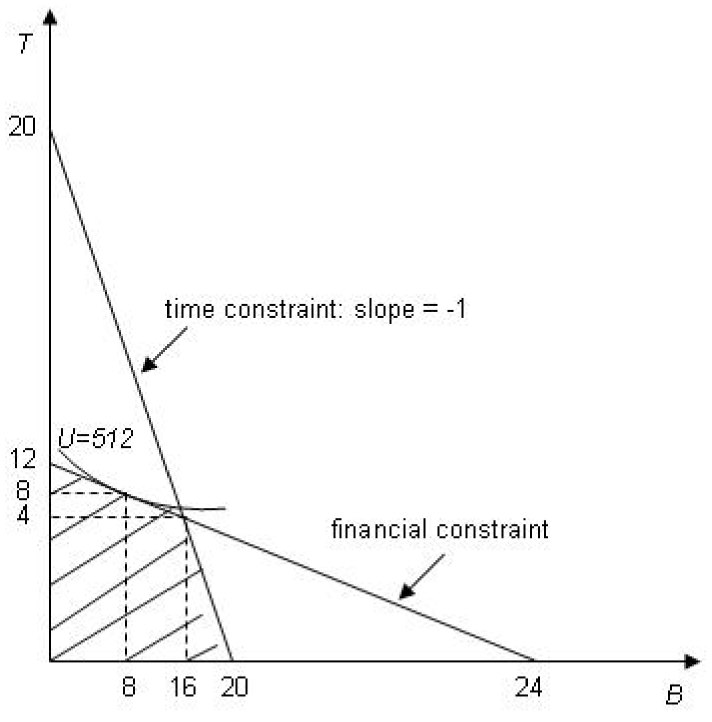
The director of the tennis grounds introduced a special offer. If you buy a monthly pass for the price of 60 zloty you pay for an hour of tennis only 20 zloty. Will Warren buy such pass and if so, how many hours of tennis and swimming will be choose per month? Provide a formal analysis and a graph.

Solution:

Financial constraint with the pass: $10B + 20T = 240$ (**1 p.**)

Time constraint: $B + T \leq 20$

So: $T = 4$, $B = 16$ (**1 p.**)



(**2 p.**)

$L = B * T^2 - \lambda(10B + 20T - 240)$

with $B \in <0,20>$ and $T \in <0,12>$

$$\frac{\partial L}{\partial B} = T^2 - 10\lambda = 0$$
$$\frac{\partial L}{\partial T} = 2TB - 20\lambda = 0 \qquad \textbf{(3 p.)}$$
$$\frac{\partial L}{\partial \lambda} = 10B + 20T - 240 = 0$$

So: $T^* = B^* = 8$

$U(8,8) = 512$
$U(8,8) > U(4,10)$ so he will buy the pass. (**1 p.**)

Problem #3
David likes strawberry and vanilla ice-cream but he prefers vanilla ice-cream stronger. David's friends laugh at him because he indicated precisely how much higher he values vanilla ice-cream in relation to strawberry ice-cream, i.e. 2 portions of vanilla ice-cream make him as well-off as 3 portions of strawberry ice-cream.

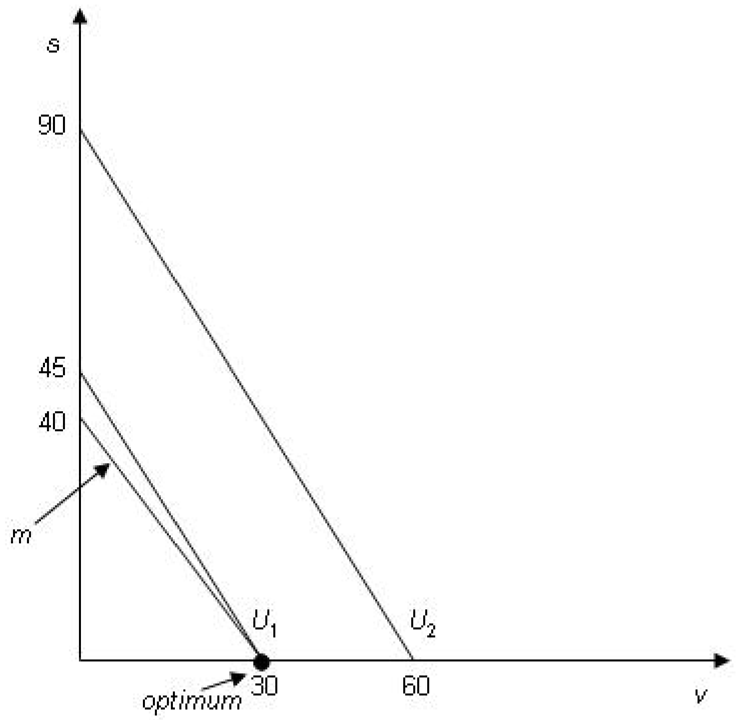
Provide David's utility function describing such preferences.

Solution:

2 portions of vanilla ice-cream (*v*) for 3 portions of strawberry ice-cream (*s*), that is $\frac{\Delta s}{\Delta v} = -\frac{3}{2}$
so $U(v,s) = 3s + 2v$ (**2 p.**)

Graph such preferences.

Solution:



(**2 p.**)

Find MRS for such preferences.

Solution:

$$\frac{\Delta s}{\Delta v} = -\frac{3}{2} = \frac{-MU_v}{-MU_s} = MRS$$ (**2 p.**)

Present David's choice if he spends 120 zloty on ice-cream per month and the prices of ice-cream are: 3 zloty per portion of strawberry ice-cream (p_s) and 4 zloty per portion of vanilla ice-cream (p_v) (calculate and present your solution on the graph above).

Solution:

$\frac{3}{4} = \frac{MU_v}{p_v} > \frac{MU_s}{p_s} = \frac{2}{3}$, so the conclusion is that he will choose to consume only vanilla ice-cream. His (optimum) choice is then: v = 120/4 = 30, s = 0. **(4 p.)**

U(v,s) = 3 * 30 + 2 * 0 = 90

/Had he chosen just strawberry ice-cream, he would have bought 120/3 = 4 portions of it and U(v,s) = 3 * 0 + 2 * 40 = 80/