Chapter Thirty-One

Welfare

Social Choice

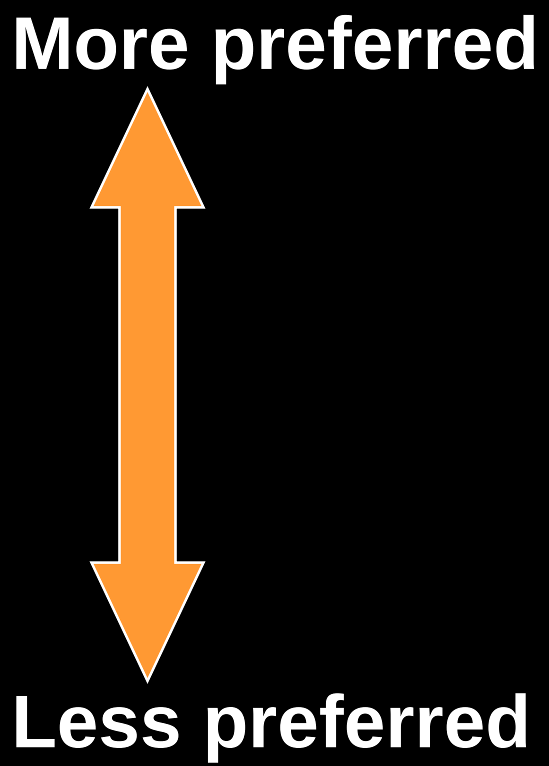
- Different economic states will be preferred by different individuals.
- How can individual preferences be "aggregated" into a social preference over all possible economic states?

Aggregating Preferences

- ***x*, *y*, *z* denote different economic states.**
- **3 agents; Bill, Bertha and Bob.**
- **Use simple majority voting to decide a state?**

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y



Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y
y beats z

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
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Majority Vote Results

x beats y
y beats z
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Aggregating Preferences

Bill	Bertha	Bob
x	y	z
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Majority Vote Results

x beats y
y beats z
z beats x

No socially best alternative!

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y
y beats z
z beats x

No socially best alternative!

Majority voting does not always aggregate transitive individual preferences into a transitive social preference.

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
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Rank-order vote results **(low score wins).**

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6
z-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6
z-score = 6

No state is selected!

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6
z-score = 6

No state is selected!

Rank-order voting is indecisive in this case.

Manipulating Preferences

- As well, most voting schemes are **manipulable.**
- I.e. one individual can cast an “untruthful” vote to improve the social outcome for himself.
- Again consider rank-order voting.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

These are truthful preferences.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
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These are truthful preferences.
Bob introduces a new alternative

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α(3)	y(3)
α(4)	x(4)	α(4)

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Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α(3)	y(3)
α(4)	x(4)	α(4)

These are truthful preferences.
Bob introduces a new alternative and then lies.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

These are truthful preferences.
Bob introduces a new alternative and then lies.

Rank-order vote results.

x-score = 8

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

These are truthful preferences.
Bob introduces a new alternative and then lies.

Rank-order vote results.

x-score = 8

y-score = 7

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

These are truthful preferences.
Bob introduces a new alternative and then lies.

Rank-order vote results.

x-score = 8
y-score = 7
z-score = 6

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

These are truthful preferences.
Bob introduces a new alternative and then lies.

Rank-order vote results.

x-score = 8 **z wins!!**
y-score = 7
z-score = 6
α-score = 9

Desirable Voting Rule Properties

- 1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
- 2. If all individuals rank x before y then so should the voting rule.
- 3. Social preference between x and y should depend on individuals' preferences between x and y only.

Desirable Voting Rule Properties

- **Kenneth Arrow's Impossibility Theorem: The only voting rule with all of properties 1, 2 and 3 is dictatorial.**

Desirable Voting Rule Properties

- **Kenneth Arrow's Impossibility Theorem:** The only voting rule with all of properties 1, 2 and 3 is dictatorial.
- Implication is that a nondictatorial voting rule requires giving up at least one of properties 1, 2 or 3.

Social Welfare Functions

- 1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
- 2. If all individuals rank x before y then so should the voting rule.
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Give up which one of these?

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- **2. If all individuals rank x before y then so should the voting rule.**
- **~~3. Social preference between x and y should depend on individuals' preferences between x and y only.~~**

Give up which one of these?

Social Welfare Functions

- 1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
- 2. If all individuals rank x before y then so should the voting rule.

There is a variety of voting procedures with both properties 1 and 2.

Social Welfare Functions

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Social Welfare Functions

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- Utilitarian: $$W = \sum_{i=1}^{n} u_i(x).$$
- Weighted-sum: $$W = \sum_{i=1}^{n} a_i u_i(x) \text{ with each } a_i > 0.$$
- Minimax: $$W = \min\{u_1(x), \ldots, u_n(x)\}.$$

Social Welfare Functions

- Suppose social welfare depends only on individuals' own allocations, instead of overall allocations.
- I.e. individual utility is $u_i(x_i)$, rather than $u_i(x)$.
- Then social welfare is
$$W = f(u_1(x_1), \ldots, u_n(x_n))$$
where f is an increasing function.

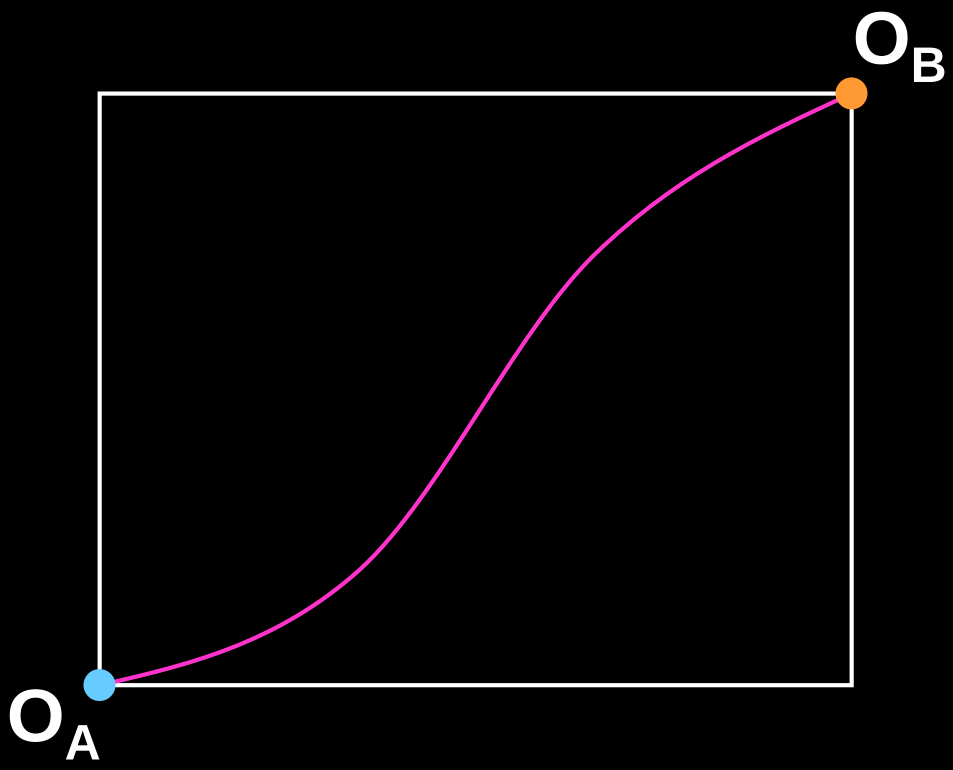
Social Optima & Efficiency

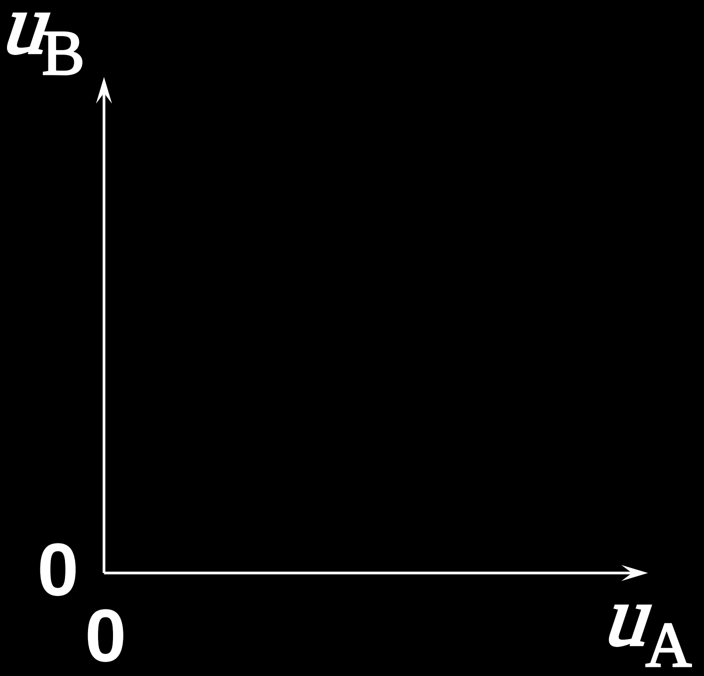
- **Any social optimal allocation must be Pareto optimal.**
- **Why?**

Social Optima & Efficiency

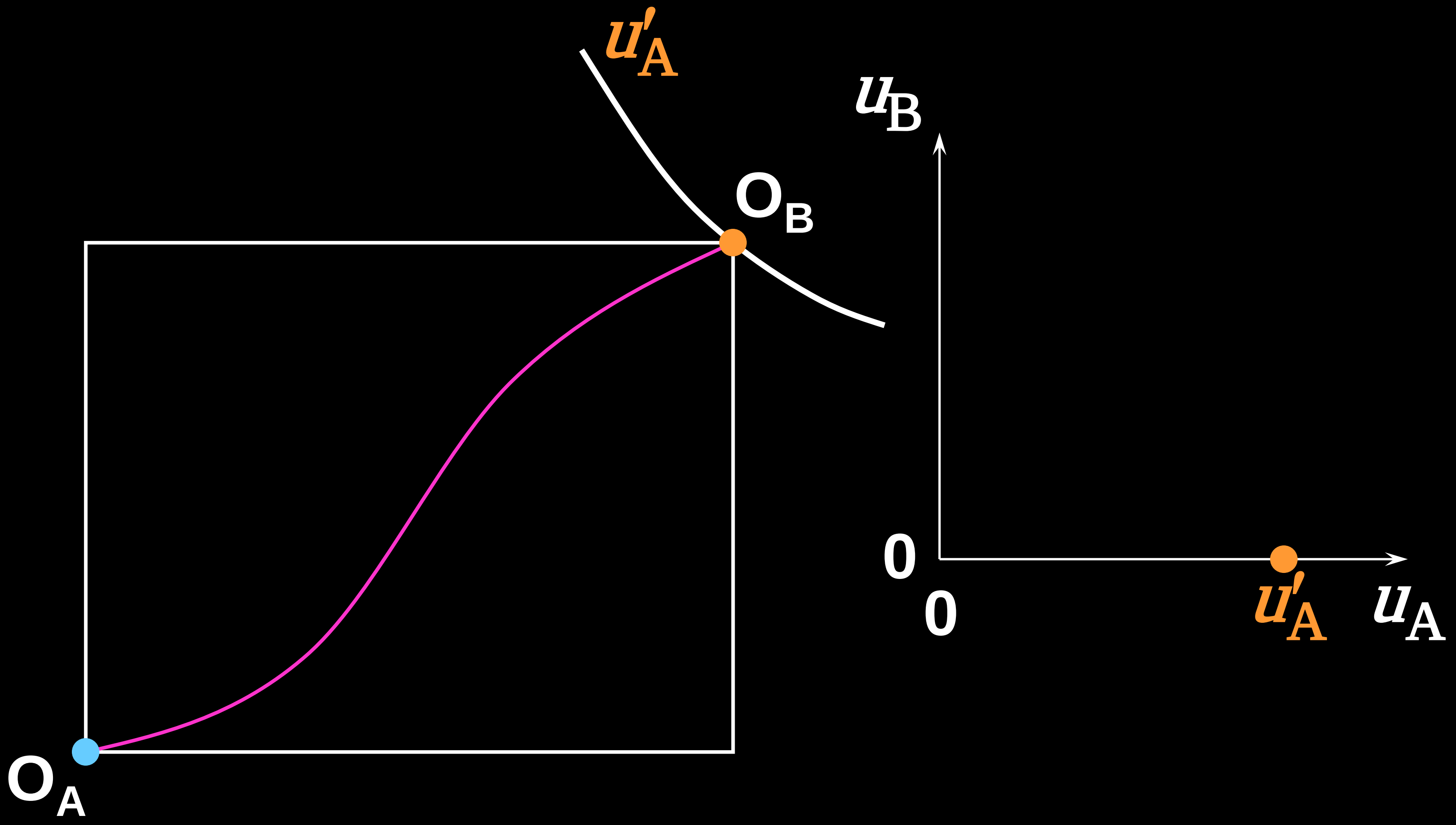
- **Any social optimal allocation must be Pareto optimal.**
- **Why?**
- **If not, then somebody's utility can be increased without reducing anyone else's utility; i.e.**
  social suboptimality $\Rightarrow$ inefficiency.

Utility Possibilities

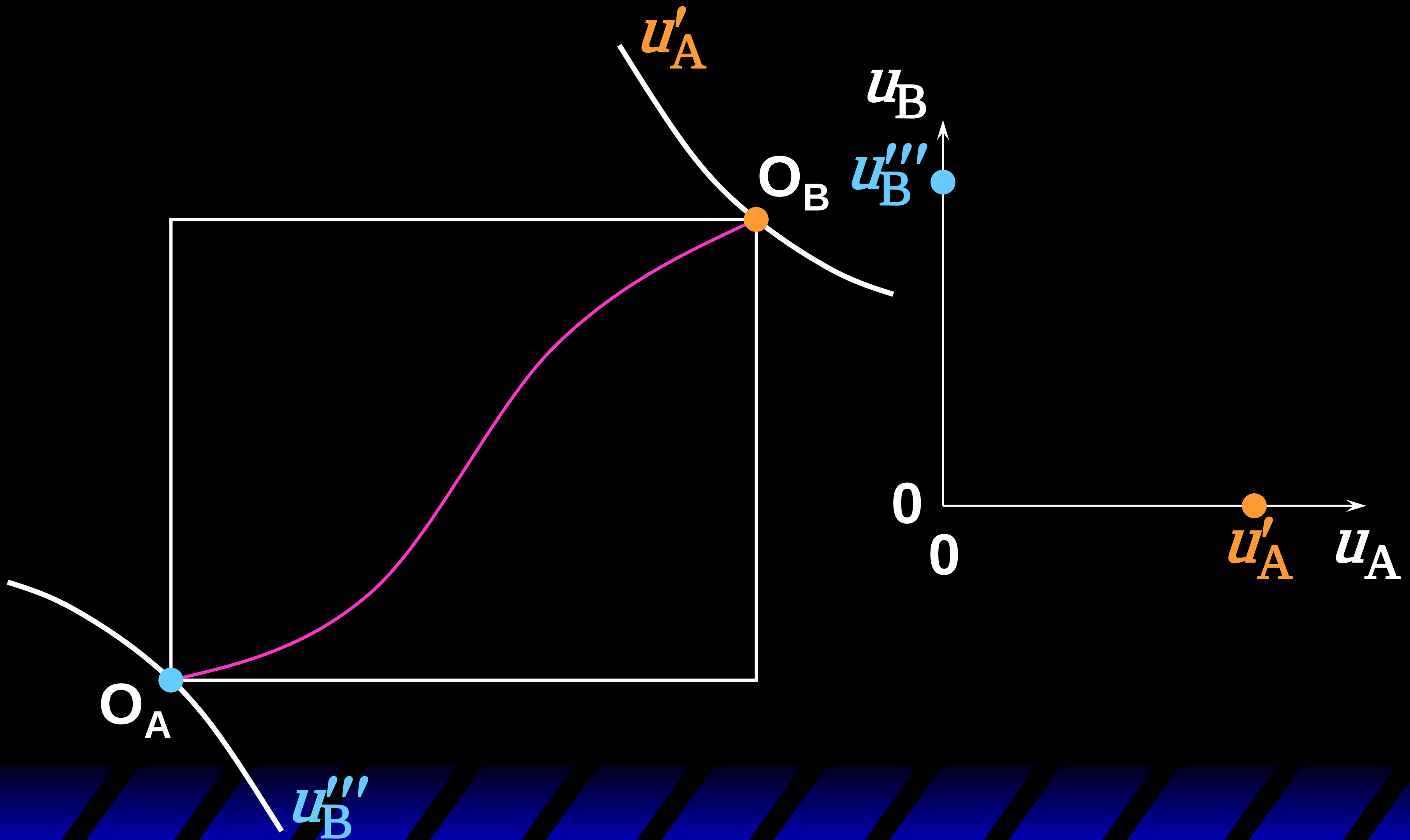




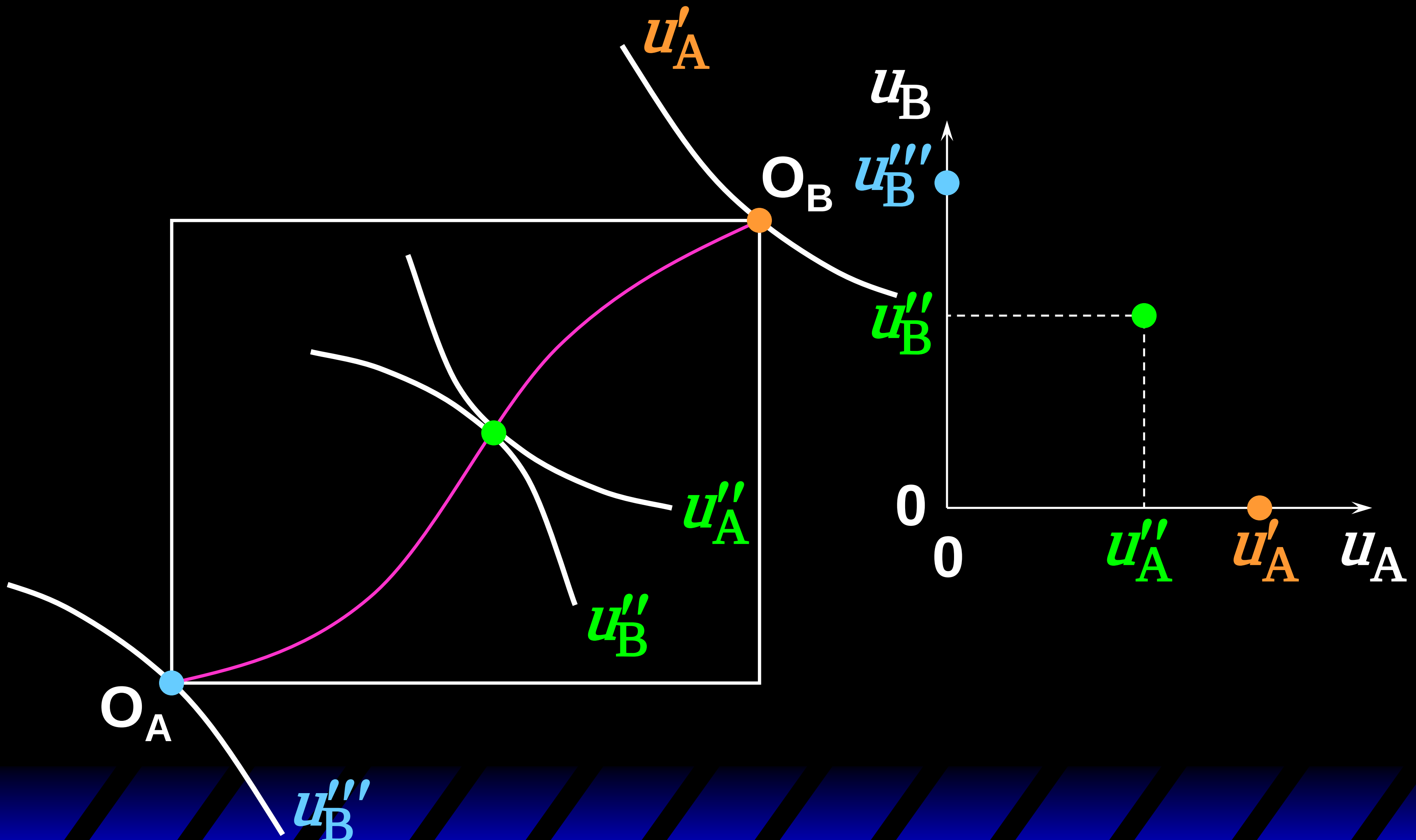
Utility Possibilities



Utility Possibilities



Utility Possibilities



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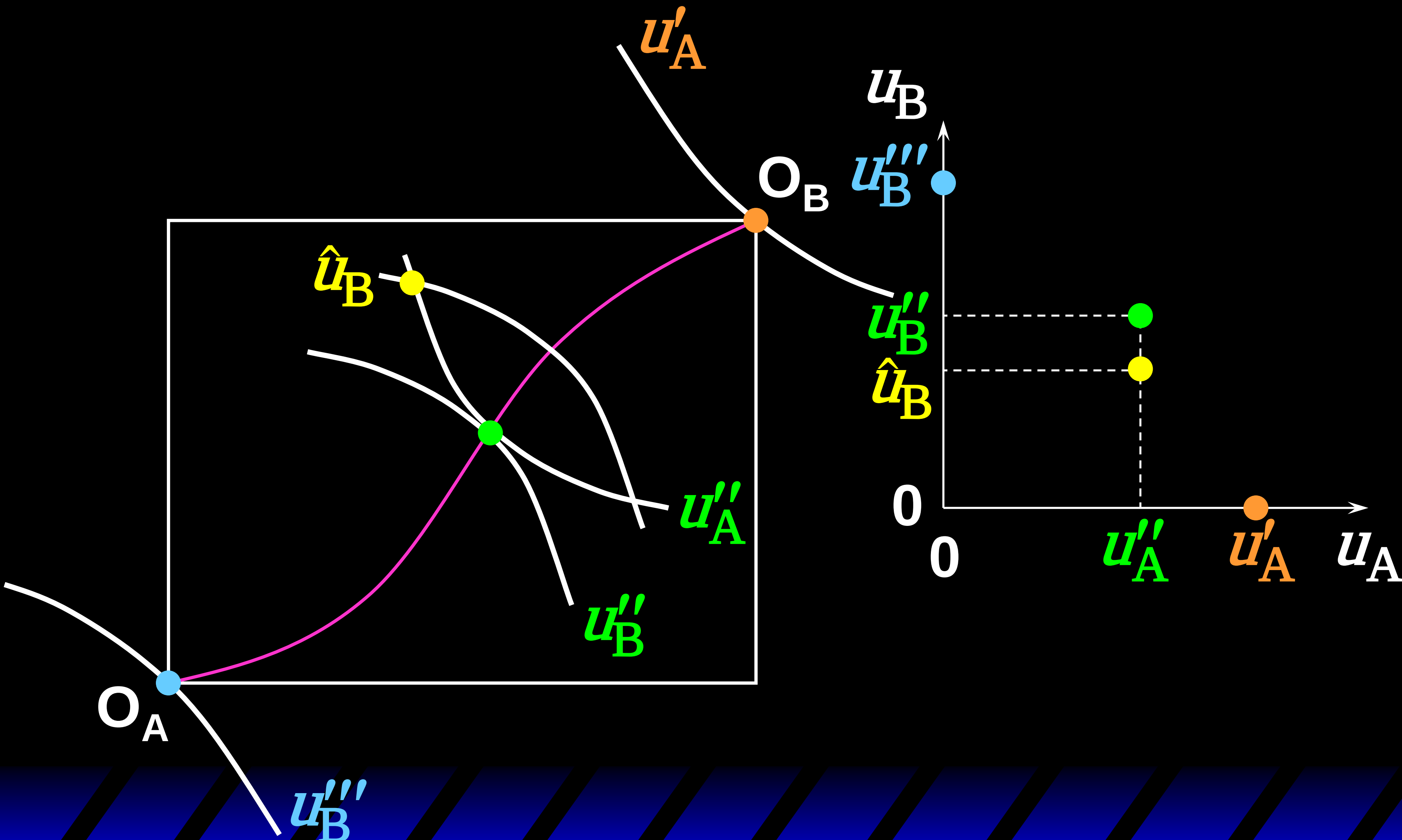

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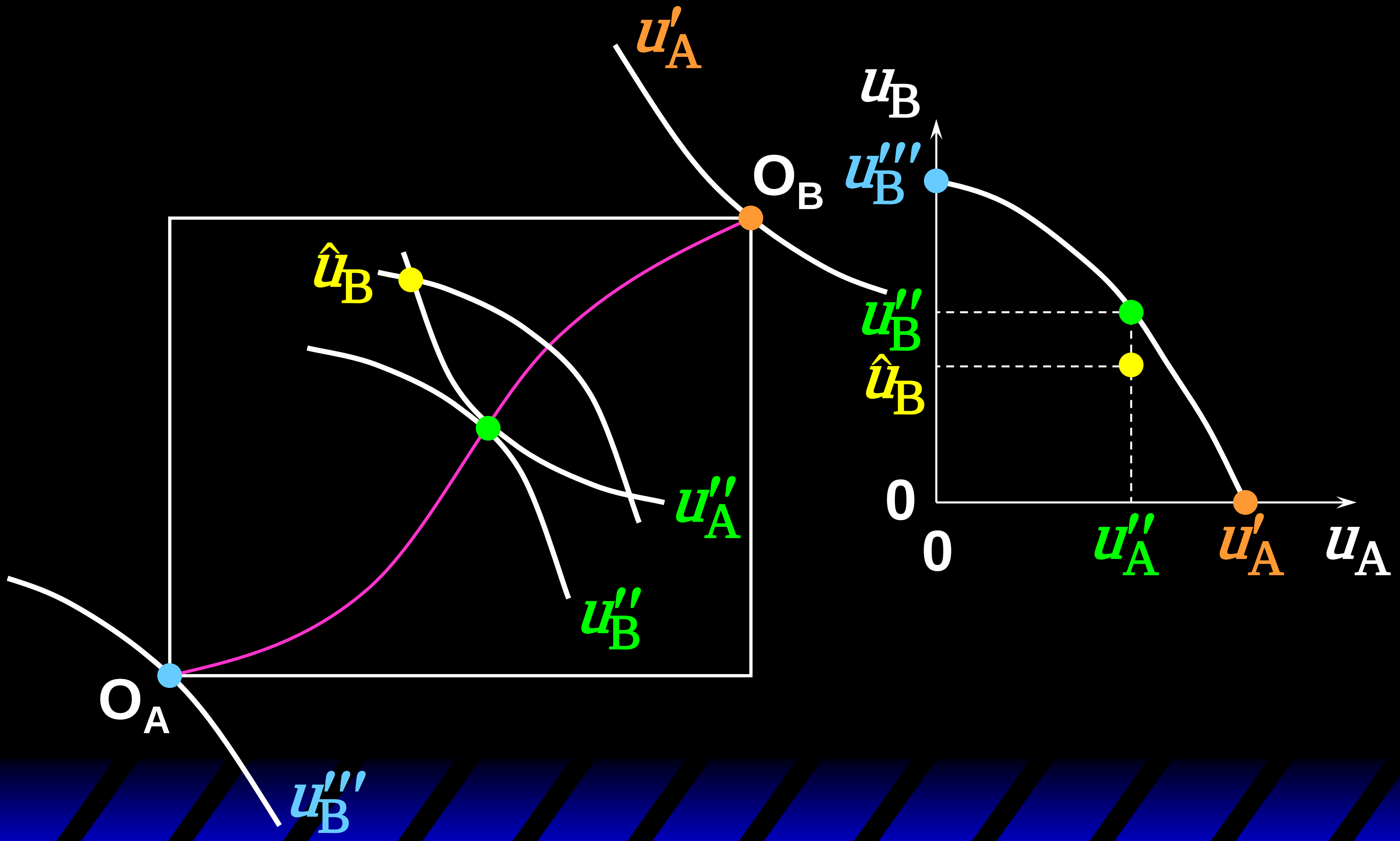

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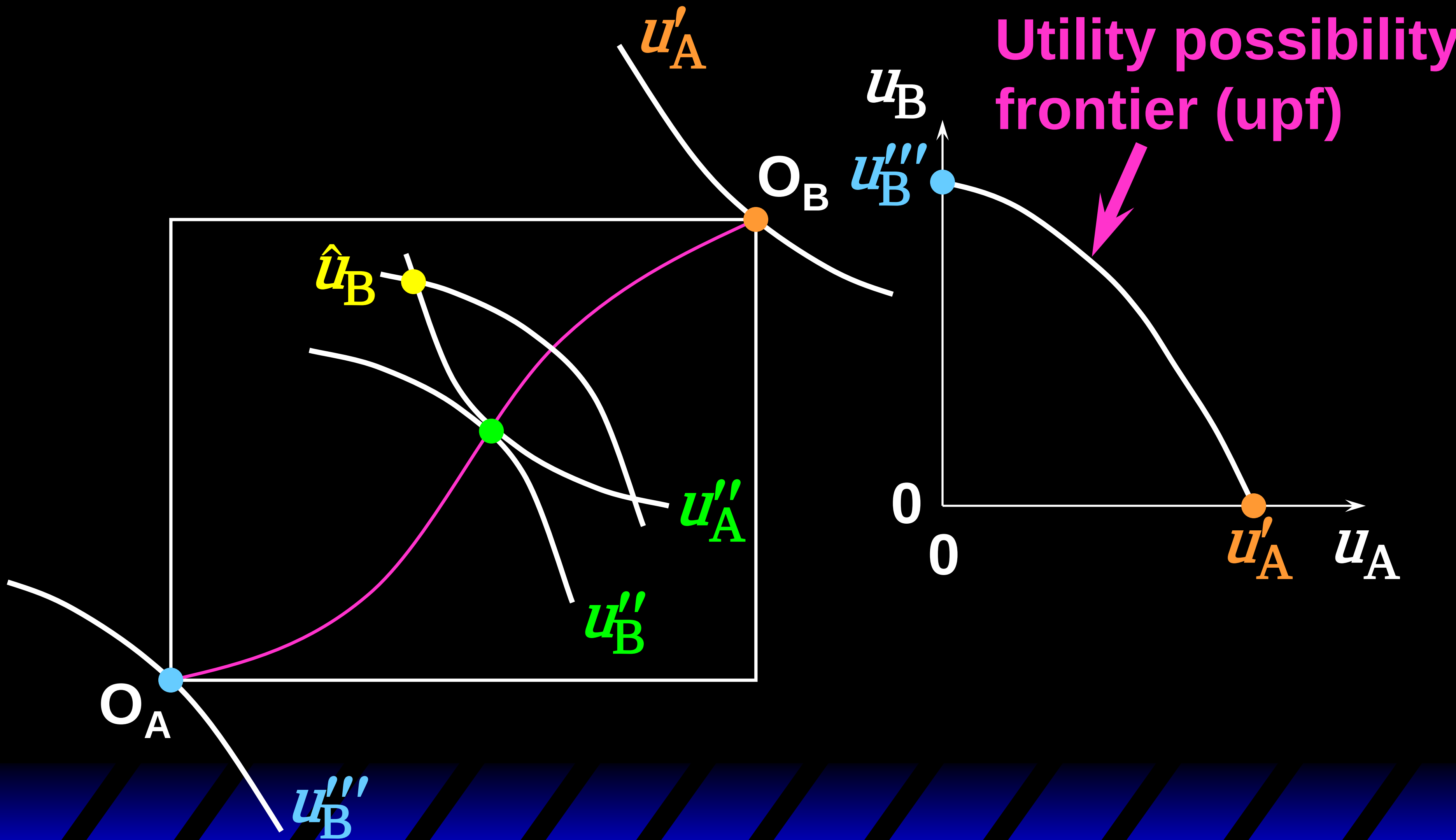

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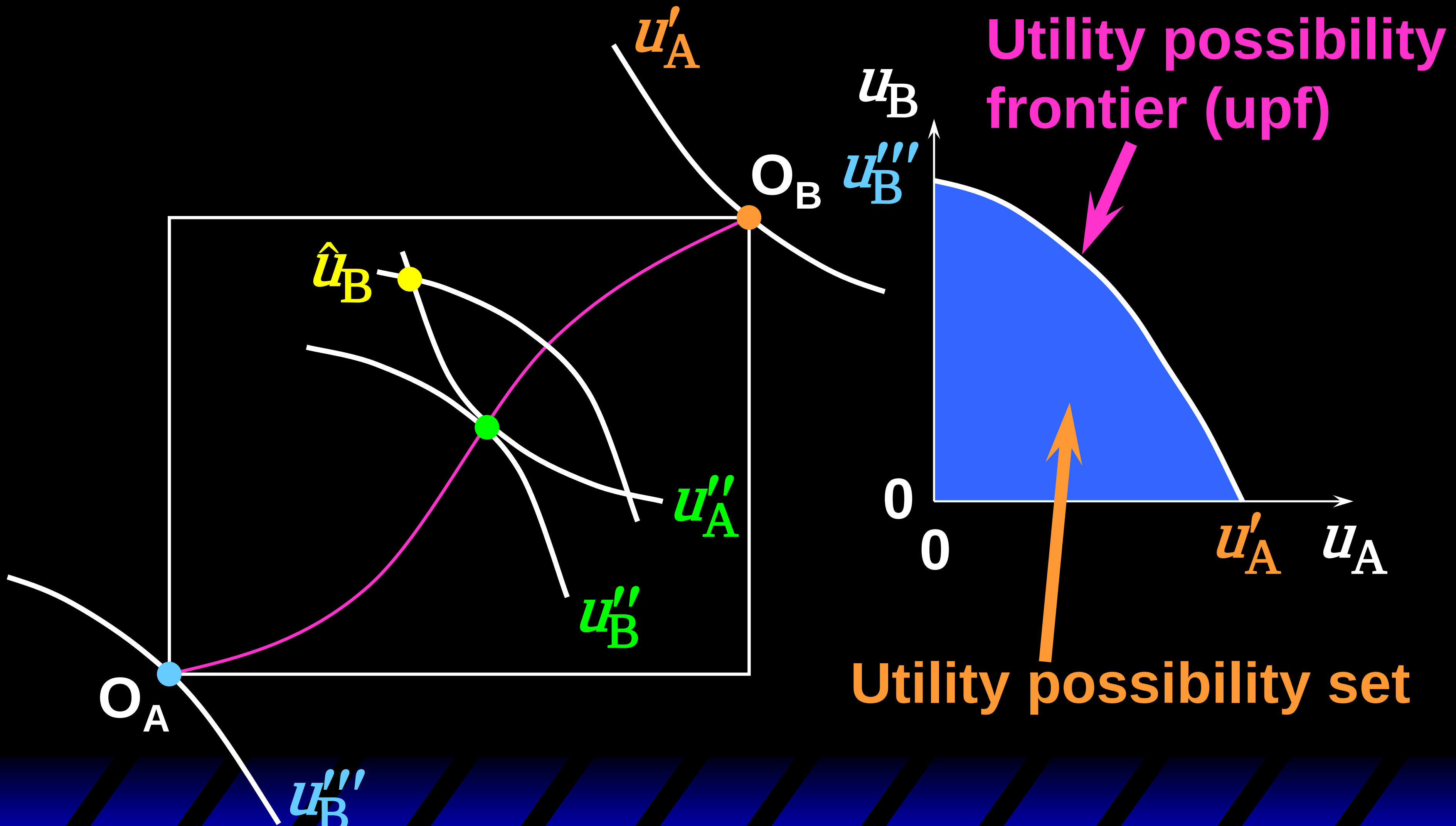
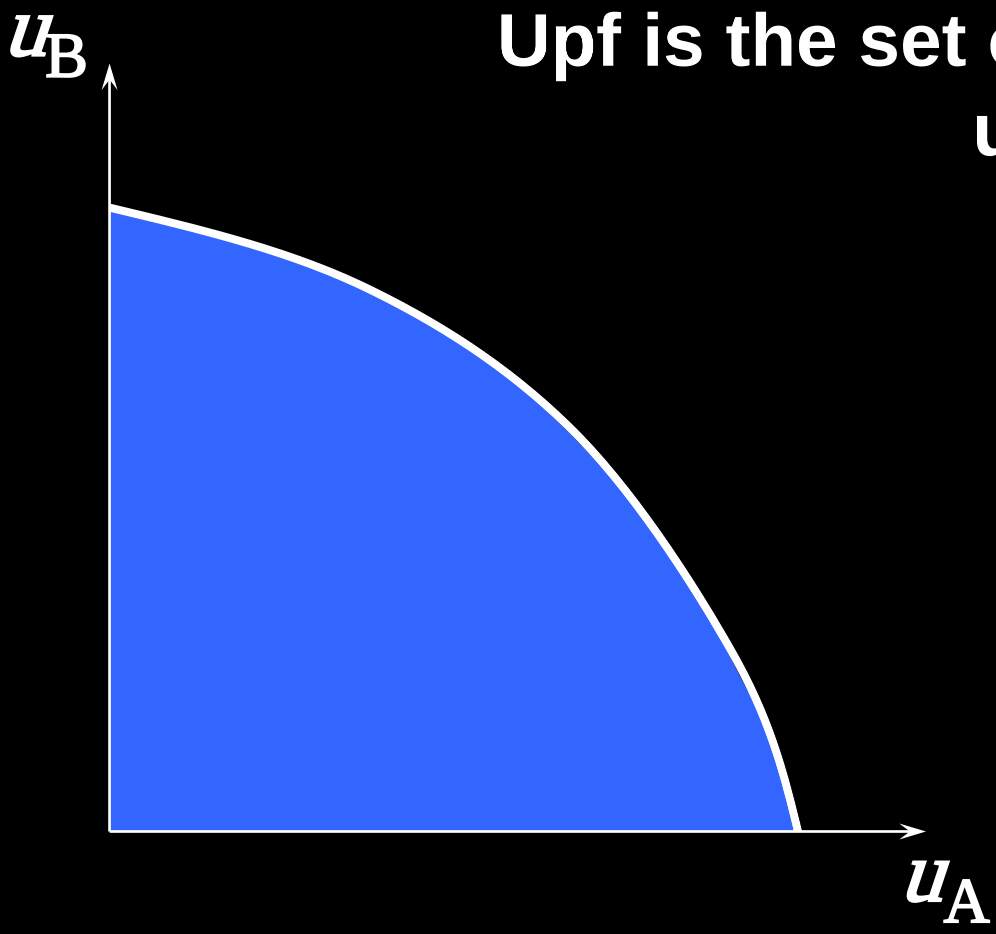
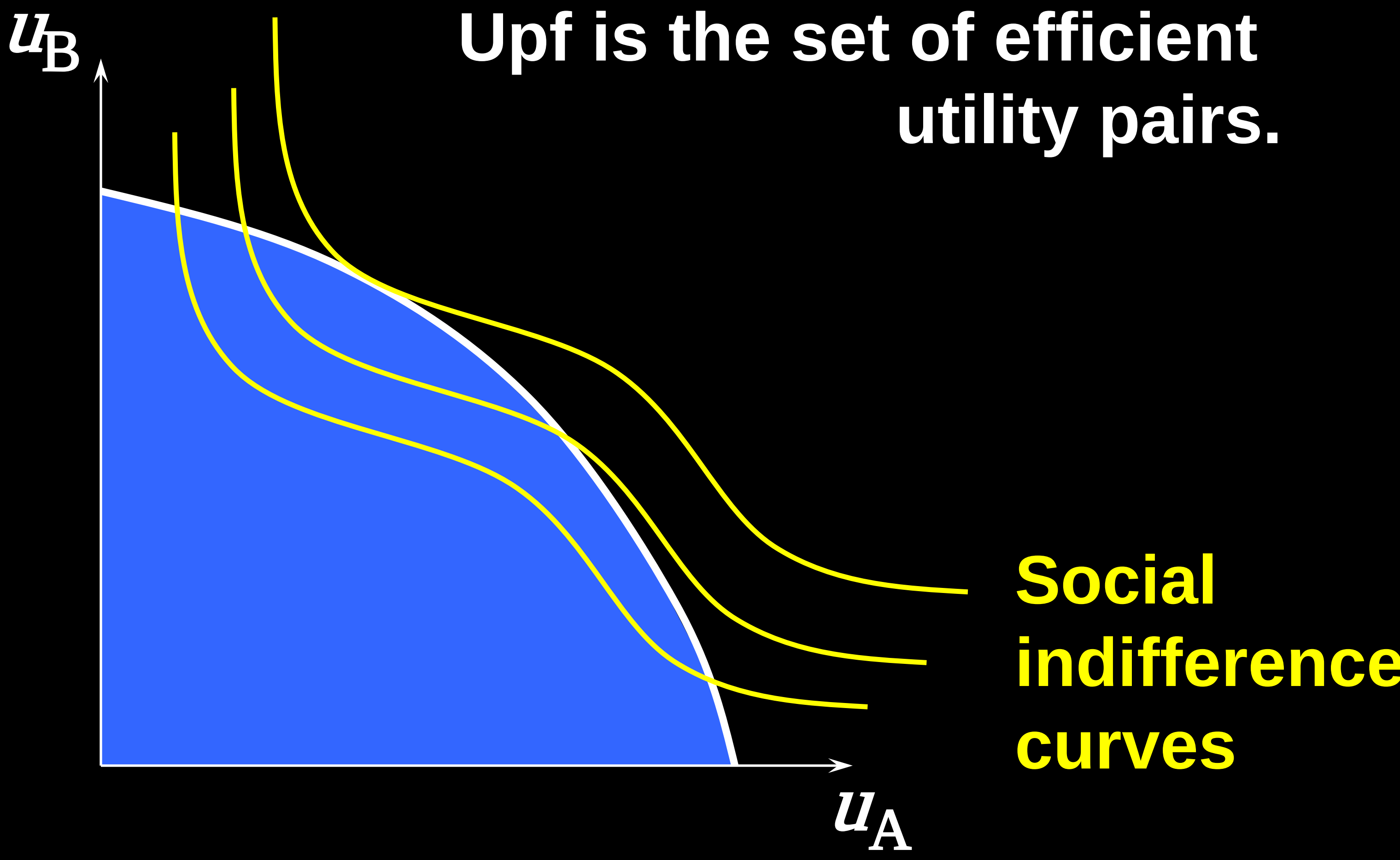


Social Optima & Efficiency

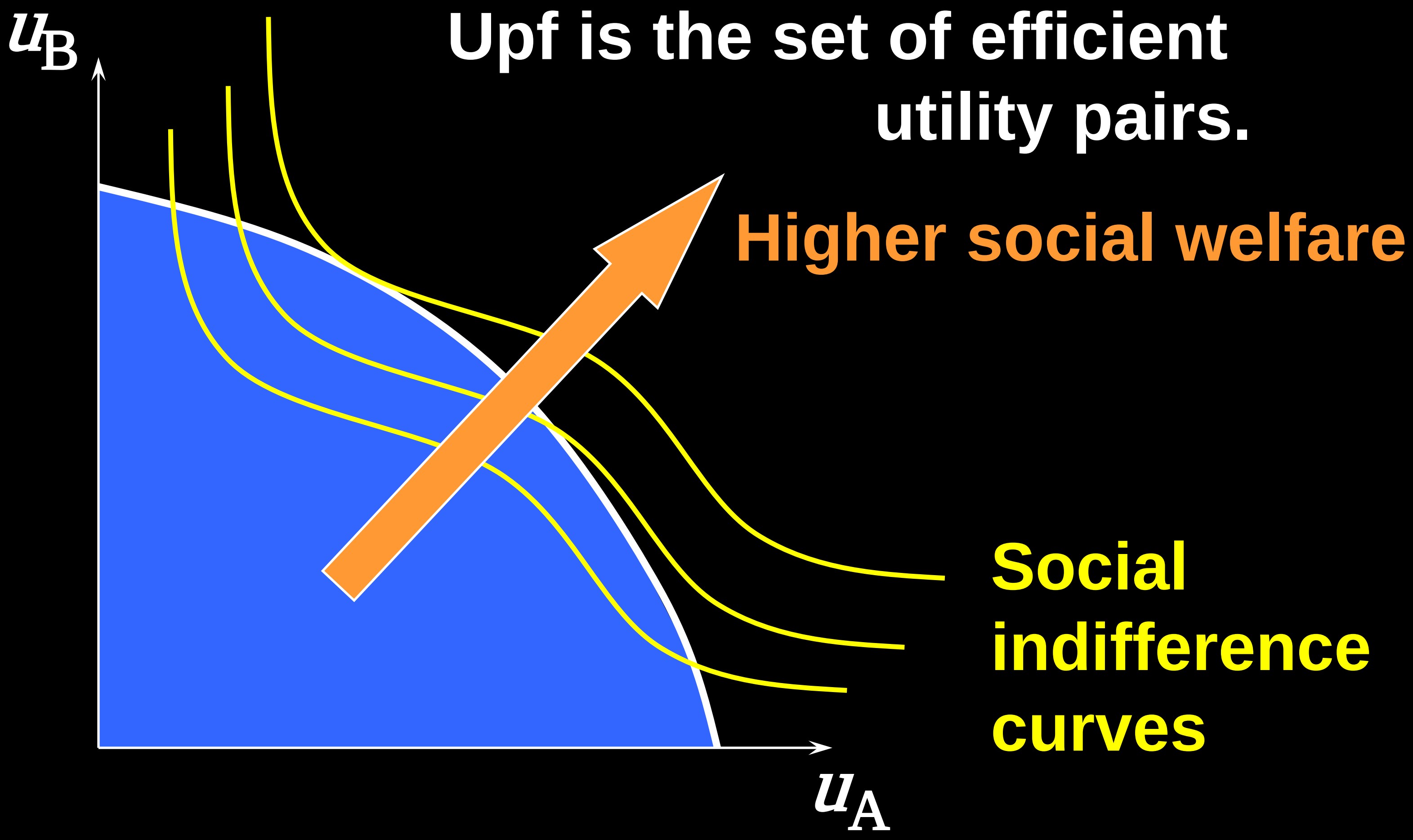
Upf is the set of efficient utility pairs.



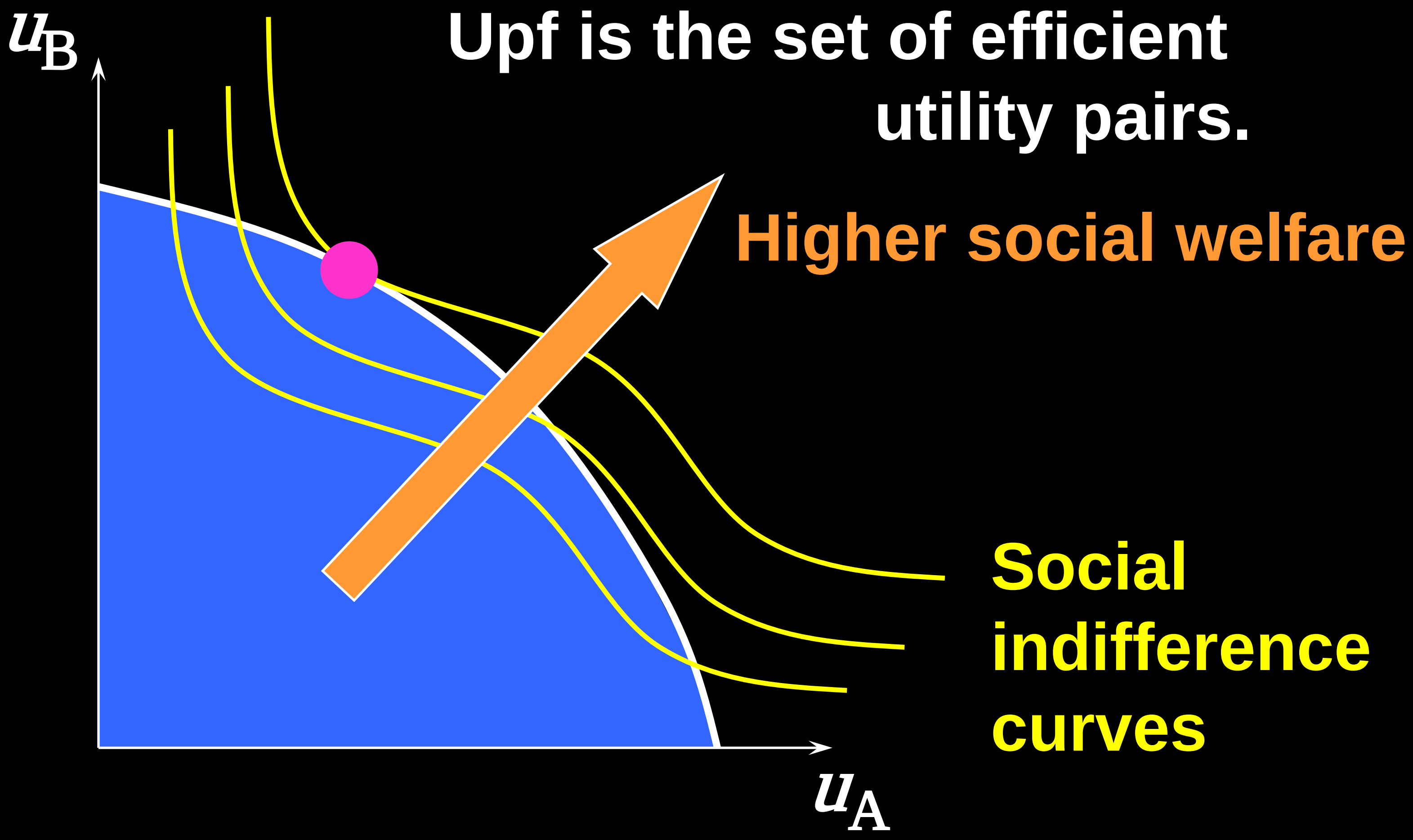
Social Optima & Efficiency



Social Optima & Efficiency



Social Optima & Efficiency



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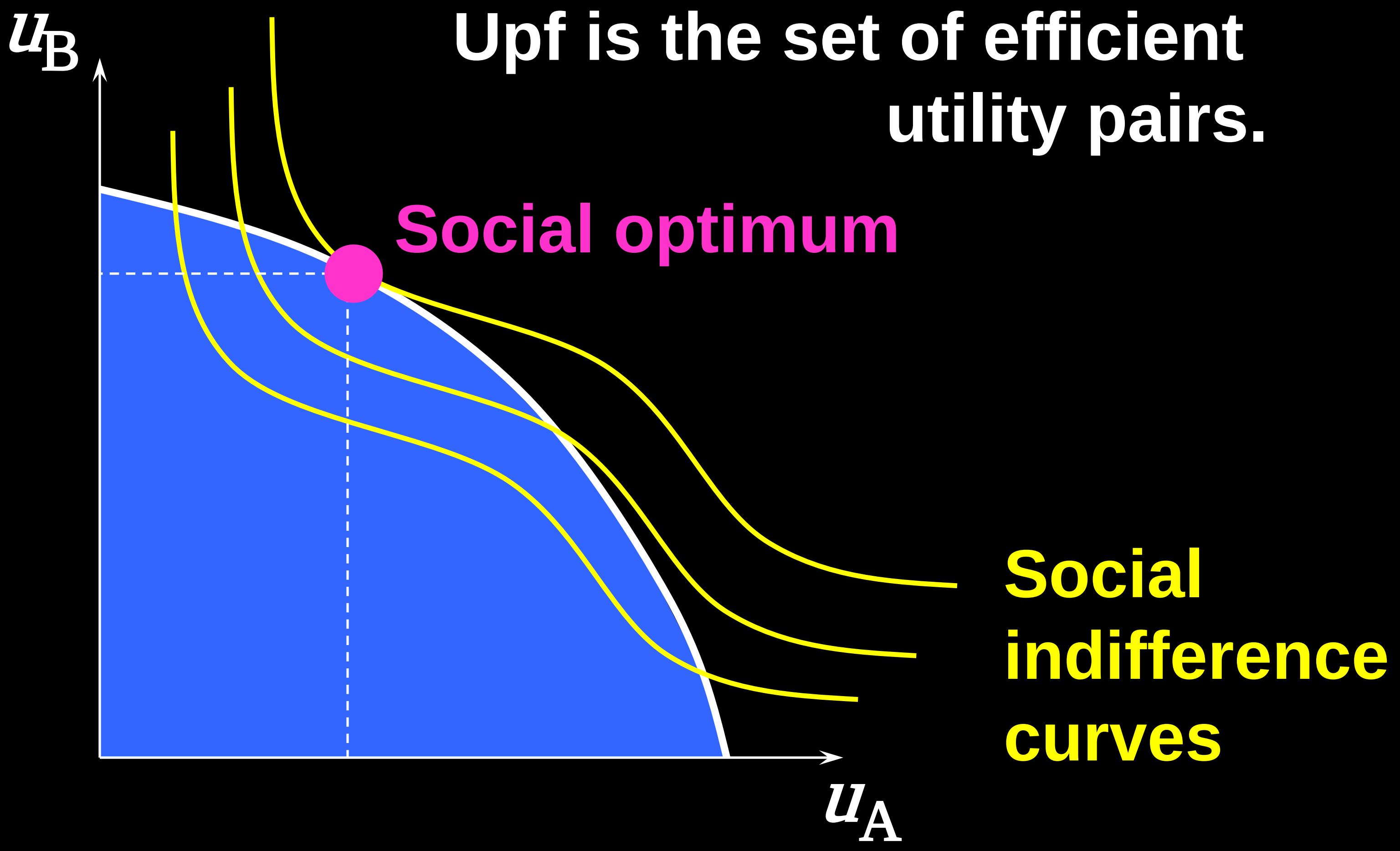
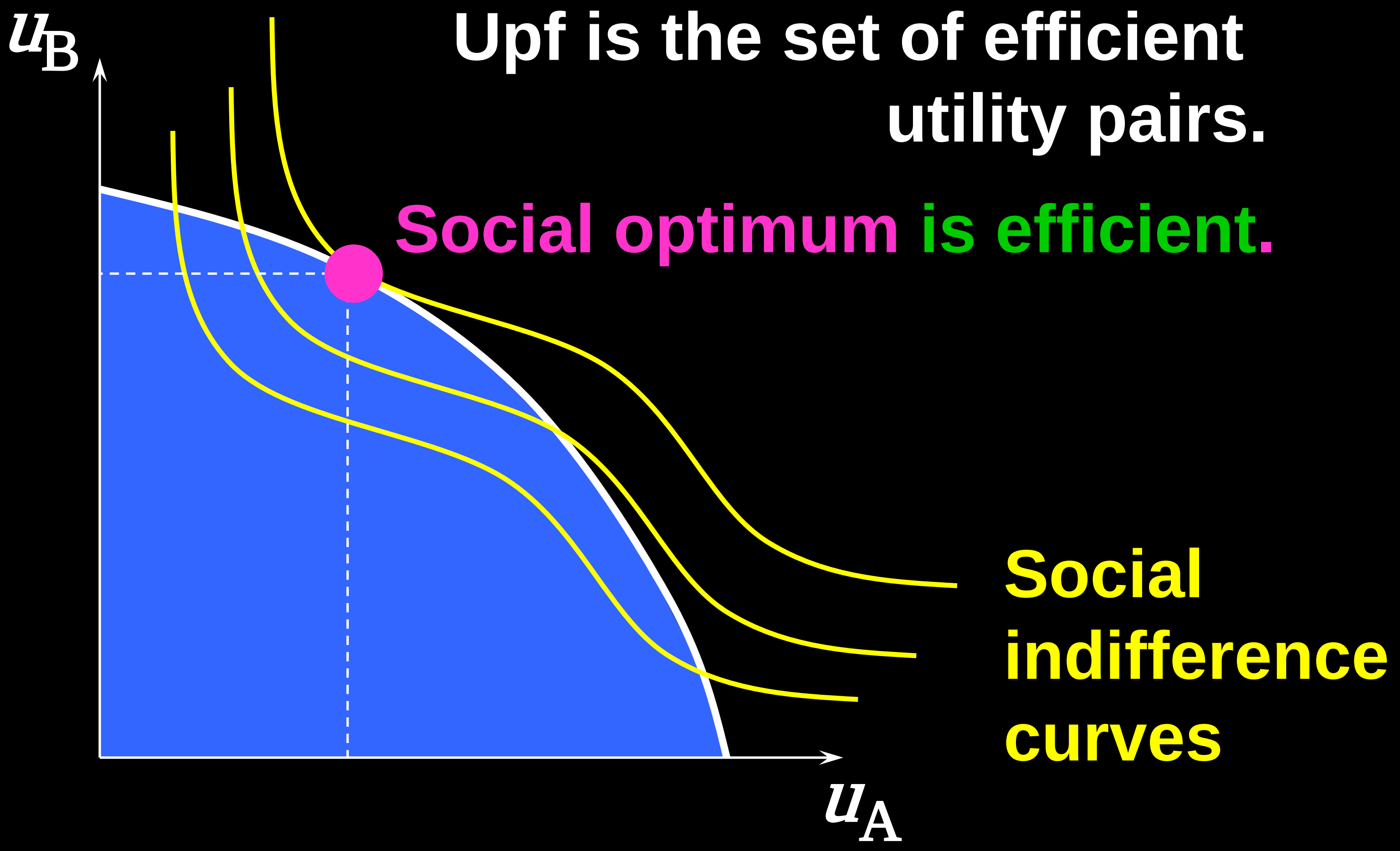


Social Optima & Efficiency



Fair Allocations

- Some Pareto efficient allocations are “unfair”.
- E.g. one consumer eats everything is efficient, but “unfair”.
- Can competitive markets guarantee that a “fair” allocation can be achieved?

Fair Allocations

- If agent A prefers agent B's allocation to his own, then agent A **envies** agent B.
- An allocation is **fair** if it is
  - Pareto efficient
  - envy free (**equitable**).

Fair Allocations

- **Must equal endowments create fair allocations?**

Fair Allocations

- Must equal endowments create fair allocations?
- No. Why not?

Fair Allocations

- 3 agents, same endowments.
- Agents A and B have the same preferences. Agent C does not.
- Agents B and C trade $\Rightarrow$ agent B achieves a more preferred bundle.
- Therefore agent A must envy agent B $\Rightarrow$ unfair allocation.

Fair Allocations

- 2 agents, same endowments.
- Now trade is conducted in competitive markets.
- Must the post-trade allocation be fair?

Fair Allocations

- 2 agents, same endowments.
- Now trade is conducted in competitive markets.
- Must the post-trade allocation be fair?
- Yes. Why?

Fair Allocations

- Endowment of each is (ω_1, ω_2).
- Post-trade bundles are

$$(x_1^A, x_2^A) \text{ and } (x_1^B, x_2^B).$$

Fair Allocations

- Endowment of each is (ω_1, ω_2).
- Post-trade bundles are
  (x_1^A, x_2^A) and (x_1^B, x_2^B).
- Then $p_1 x_1^A + p_2 x_2^A = p_1\omega_1 + p_2\omega_2$

  and $p_1 x_1^B + p_2 x_2^B = p_1\omega_1 + p_2\omega_2$.

Fair Allocations

- **Suppose agent A envies agent B.**
- **I.e.** $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A).$

Fair Allocations

- **Suppose agent A envies agent B.**
- **I.e.** $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A).$
- **Then, for agent A,**

$$p_1 x_1^B + p_2 x_2^B > p_1 x_1^A + p_2 x_2^B$$
$$= p_1 \omega_1 + p_2 \omega_2.$$

Fair Allocations

- Suspect agent A envies agent B.
- I.e. $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- Then, for agent A,

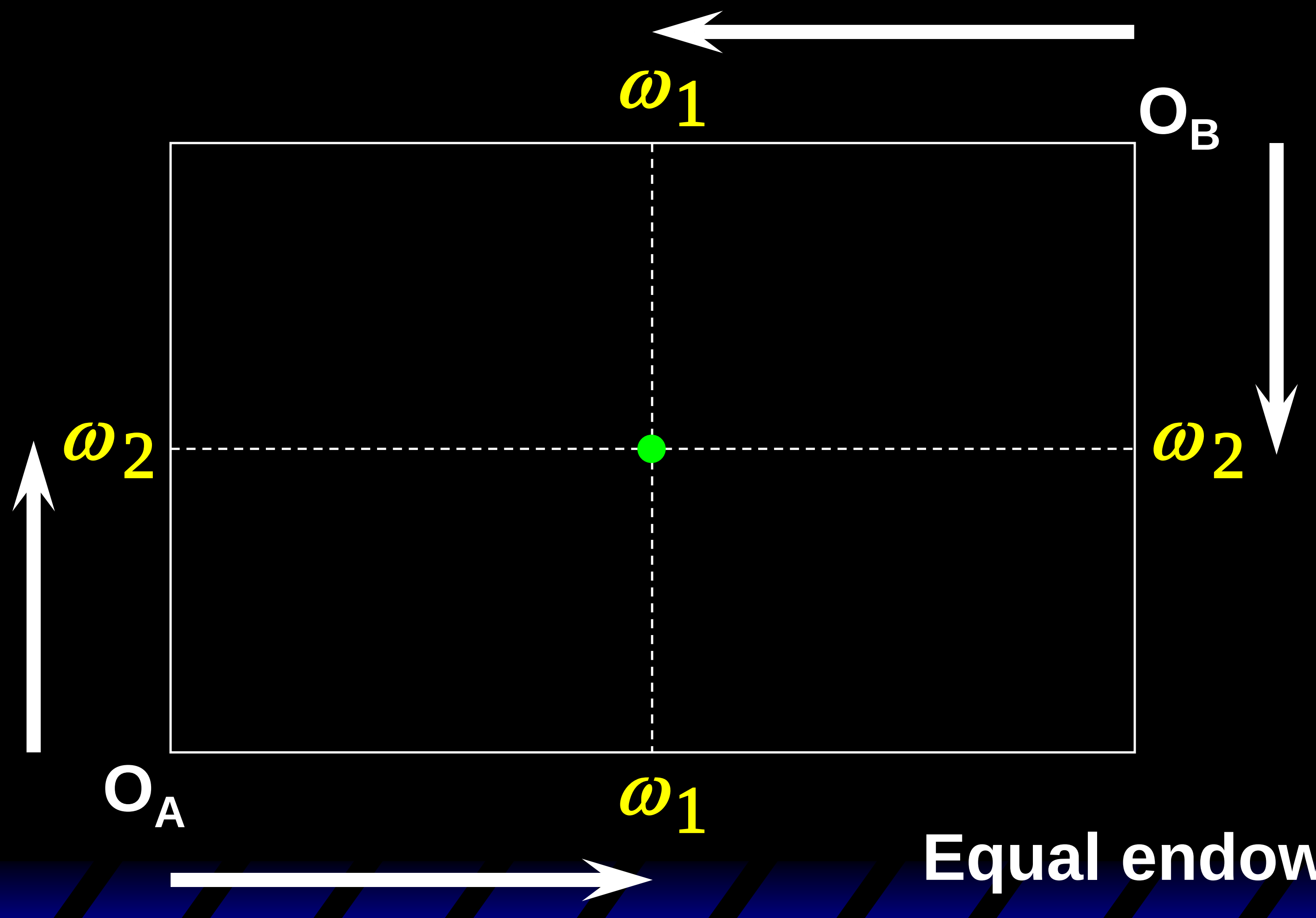
$$p_1 x_1^B + p_2 x_2^B > p_1 x_1^A + p_2 x_2^B$$
$$= p_1 \omega_1 + p_2 \omega_2.$$

- Contradiction. (x_1^B, x_2^B) is not affordable for agent A.

Fair Allocations

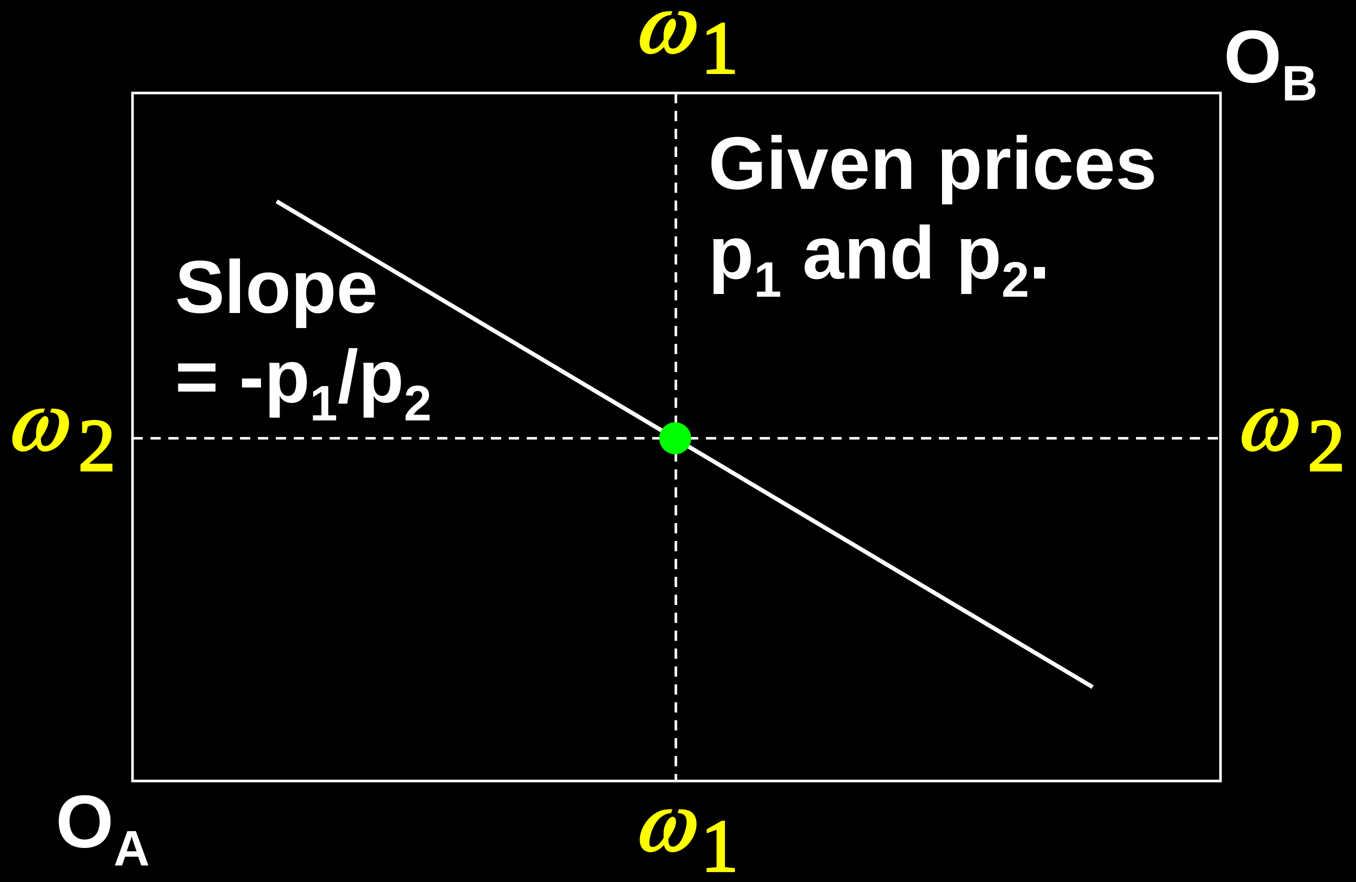
- **This proves: If every agent's endowment is identical, then trading in competitive markets results in a fair allocation.**

Fair Allocations

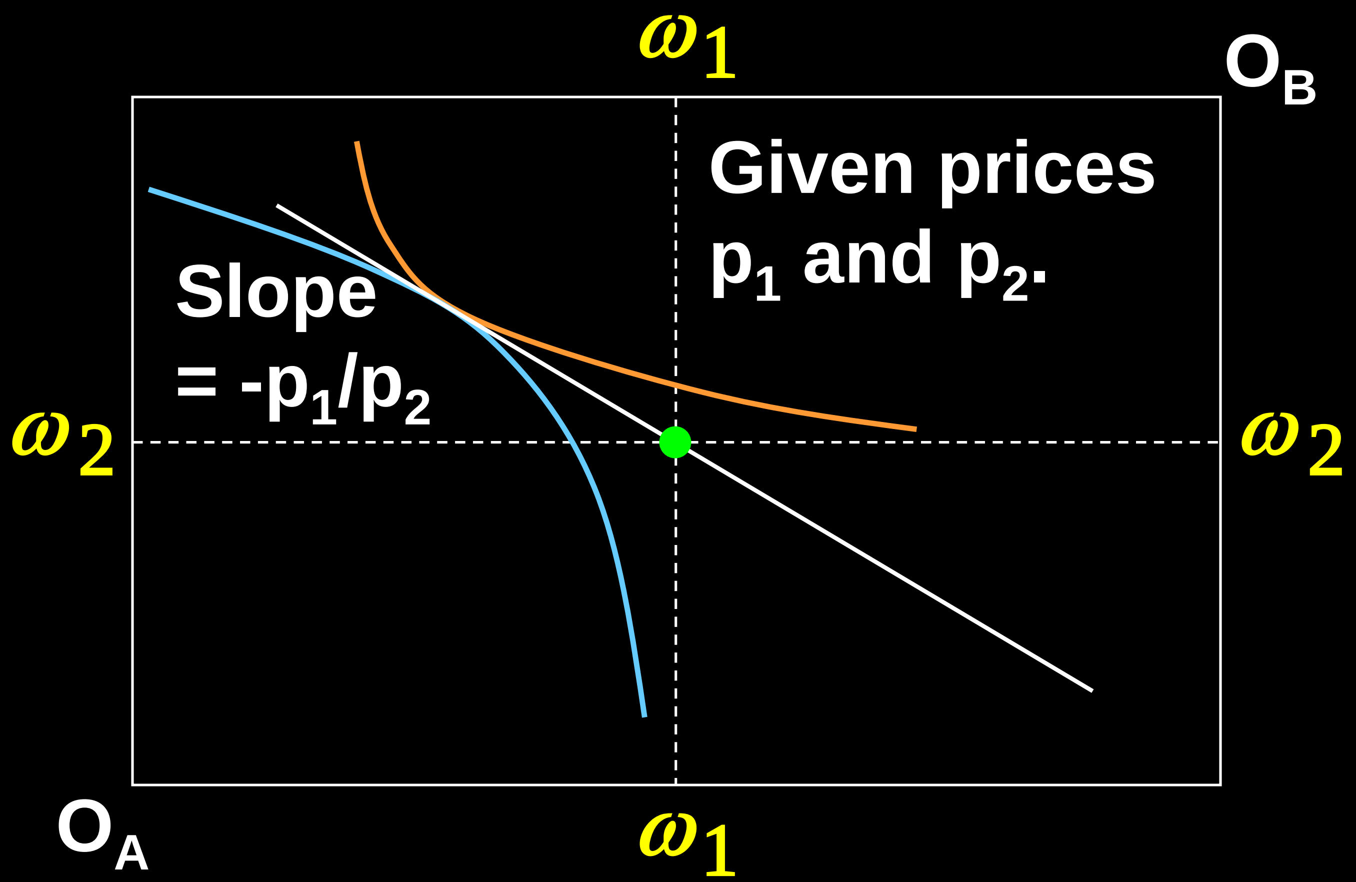


Equal endowments.

Fair Allocations



Fair Allocations



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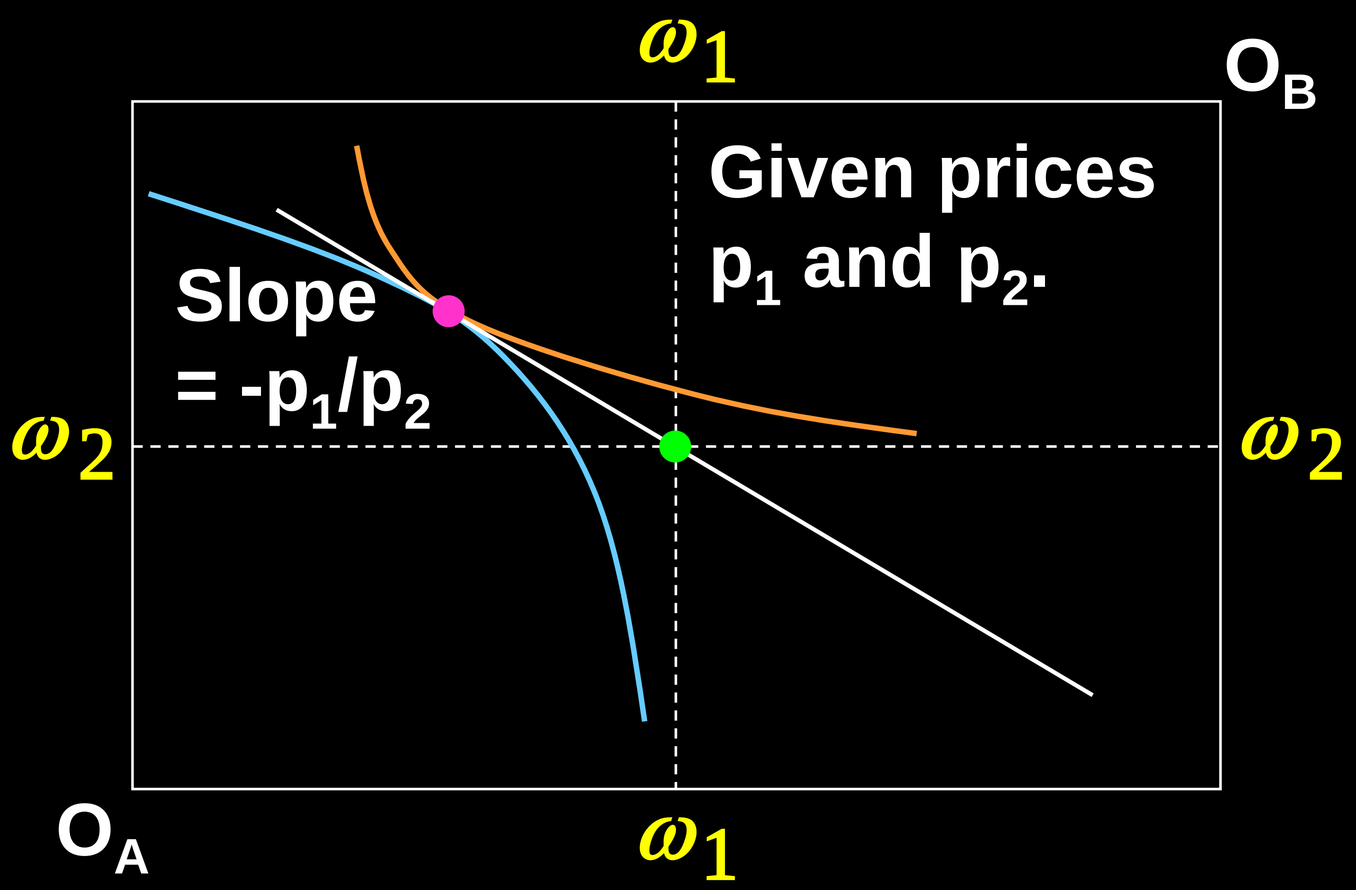
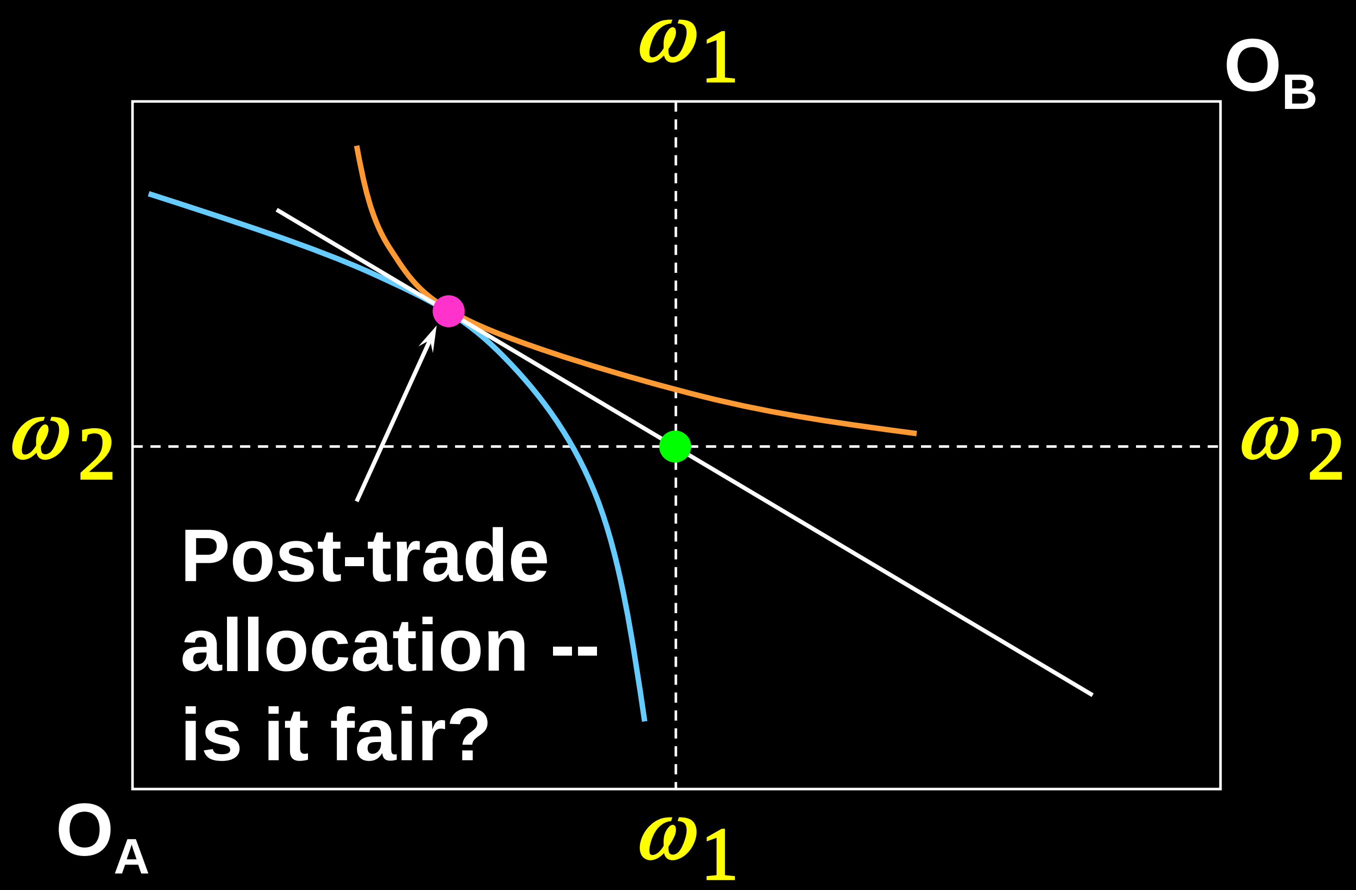


Fair Allocations



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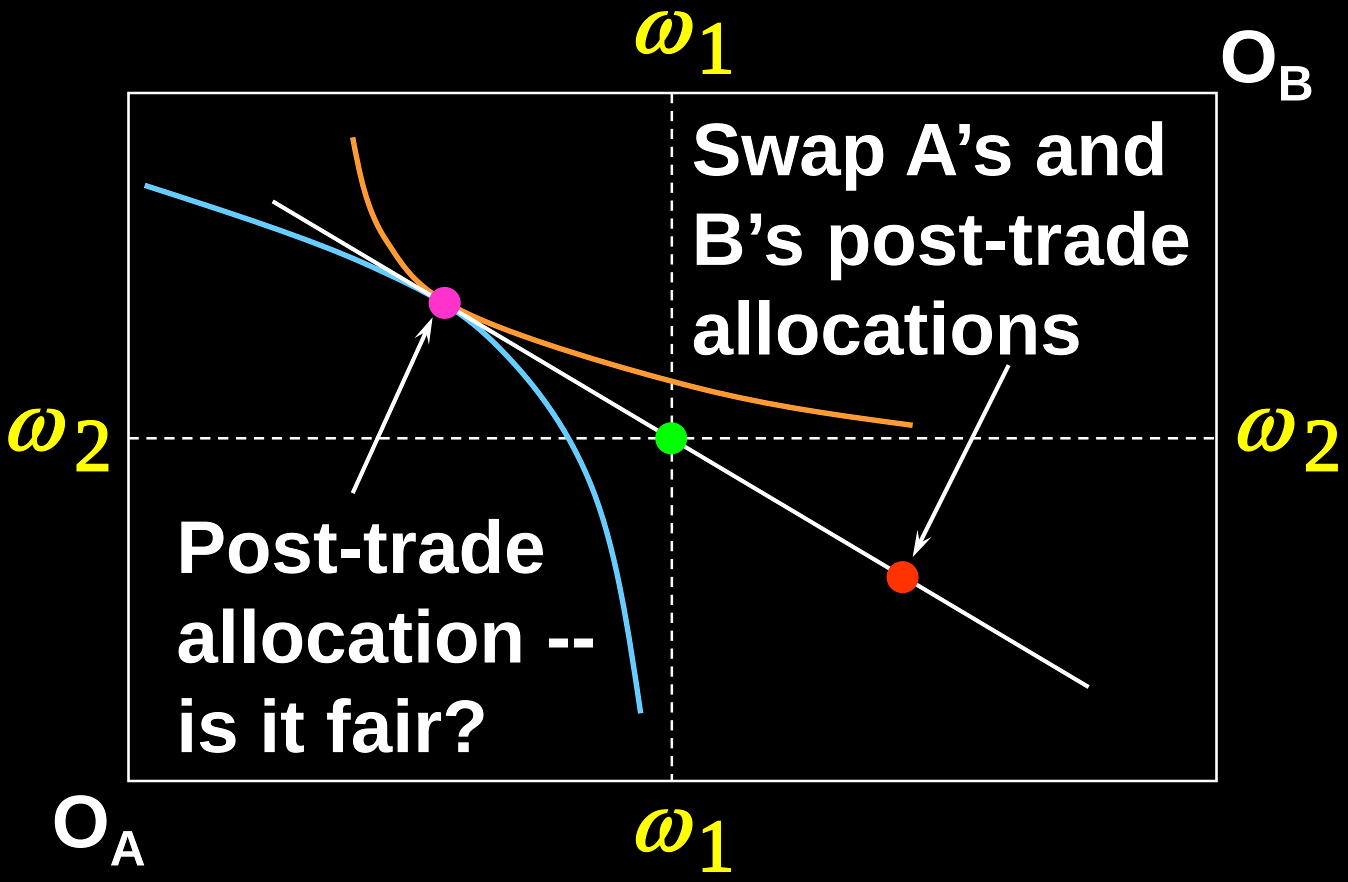

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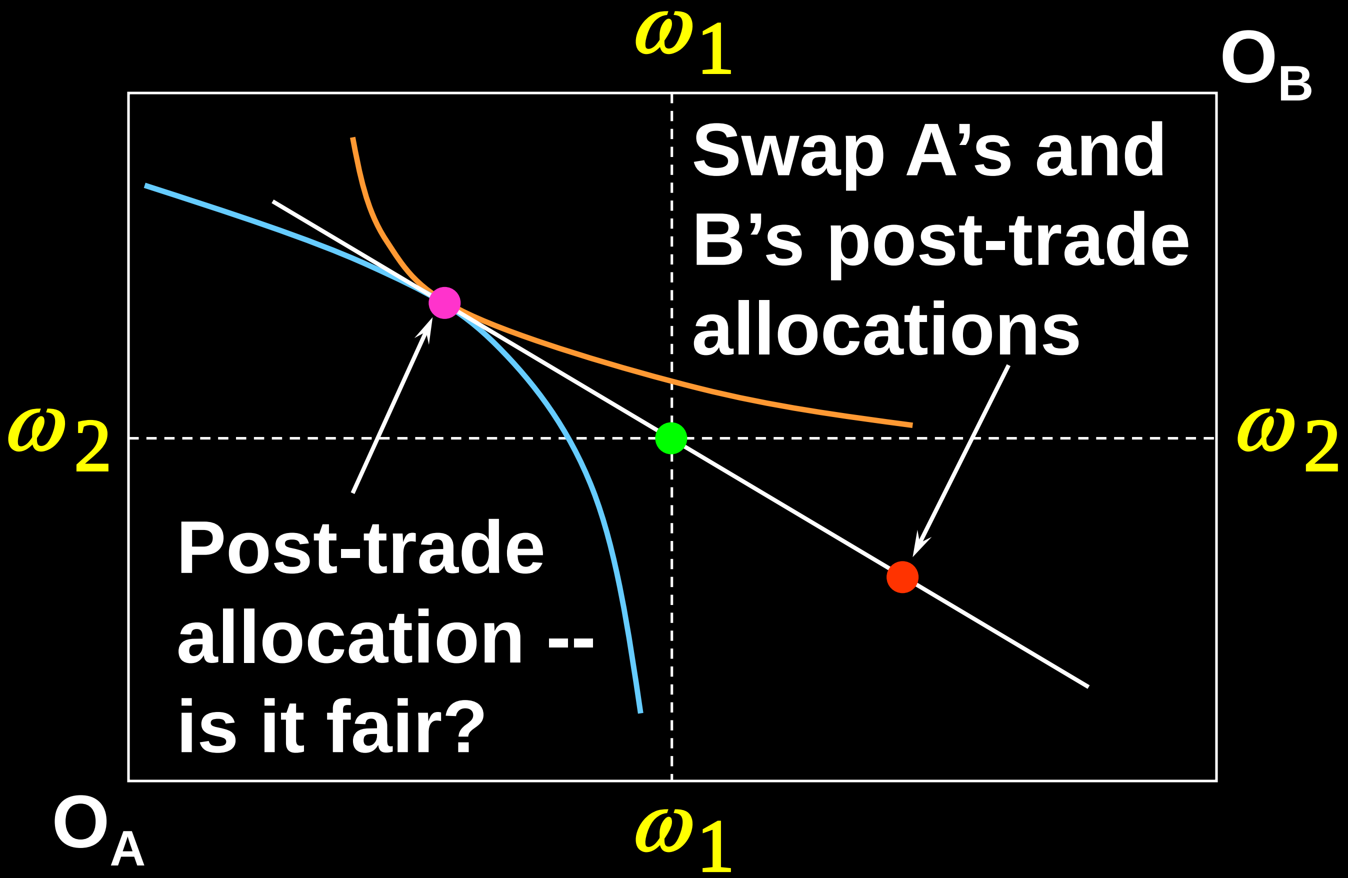
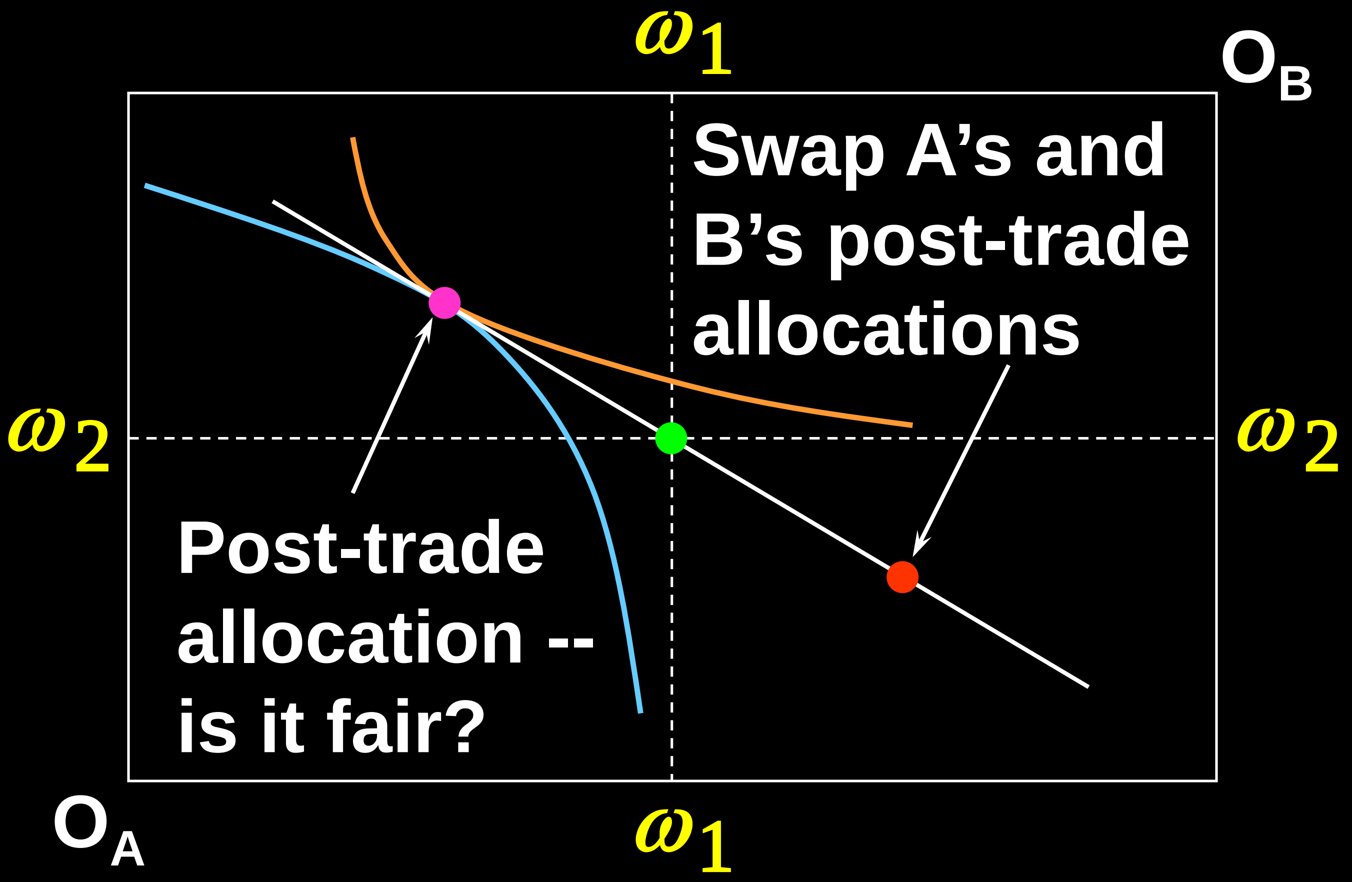


A does not envy B's post-trade allocation.
B does not envy A's post-trade allocation.

Fair Allocations



Post-trade allocation is Pareto-efficient and envy-free; hence it is fair.