MICROECONOMICS 3
PROBLEMS #3

PURE EXCHANGE, GENERAL EQUILIBRIUM

Problem #1

Are the following statements true or false? Provide a reasoning:

a) If an allocation is Pareto-efficient, then there exists no such allocation, where one of the participants of the exchange has lower utility.
b) If the Pareto-efficient allocation has been reached, then it is impossible to increase utility of any of the participants of the exchange.
c) If MU of each good is always positive for both participants of the exchange, then the situation, when one of them has nothing, is Pareto-efficient.
d) There exists such Pareto-efficient allocation, where each participant's utility is higher than in another Pareto-efficient allocation.
e) For every equilibrium allocation it holds that the price ratio is equal to the marginal rate of substitution for both participants of the exchange.
f) Within the Edgeworth box the Pareto set is always a certain curve, where a given allocation of one good corresponds to precisely one allocation of the other good.

Problem #2

For the utility functions of two persons functioning in a pure exchange model that are given below:

a) graph the Edgeworth box for initial endowments ω_{1x}, ω_{1y}, ω_{2x}, ω_{2y}. In your graphs try to avoid situations where the initial allocation (endowment) is located on one of the diagonals of the box;
b) graph the indifference curves passing through the initial endowment point;
c) depict the area composed by points which represent improvement for both consumers relative to their initial situation;
d) find the formula for the contract curve;
e) graph the contract curve.

i. $U_1(x,y) = xy$, $U_2(x,y) = x^{0.5}y^{1.5}$
ii. $U_1(x,y) = 4x + 2y$, $U_2(x,y) = x + y$
iii. $U_1(x,y) = xy$, $U_2(x,y) = x + 3y$
iv. $U_1(x,y) = x + 2y$, $U_2(x,y) = \min\{2x,y\}$
v. $U_1(x,y) = x^2y^{0.5}$, $U_2(x,y) = \min\{x,2y\}$

Assuming that $\omega_{1x} = 10$, $\omega_{1y} = 10$, $\omega_{2x} = 20$, $\omega_{2y} = 20$:

1) graph the Edgeworth box;
2) find the demand of both involved persons for each of the goods;
3) find the equilibrium price ratio

for all points (i)-(viii) given above

Problem #3

Paul's utility function is given by the formula $U_P(x_{P1}, x_{P2}) = x_{P1}^{1/3} x_{P2}^{2/3}$ and John's utility function is $U_J(x_{J1}, x_{J2}) = x_{J1}^{1/2} x_{J2}^{1/2}$. Paul disposes of an initial endowment $\omega_{P1} = 1$ and $\omega_{P2} = 0$, while for John it is $\omega_{J1} = 0$ and $\omega_{J2} = 1$. Their only income comes from selling some of the initial endowments. Assuming that prices p_1 and p_2 of goods x_1 and x_2 are determined by an impartial arbiter-auctioneer, at what ratio of these prices will the equilibrium for the exchange between Paul and John take place?

Multiple-choice questions:

Problem #1

In a pure exchange economy consumers A and B exchange goods *x* and *y*. The utility function of consumer A takes the form $U_A(x_A,y_A) = x_A y_A$, while the utility function of consumer B is $U_B(x_B,y_B) = 3x_B+2y_B$. Consumer A's initial endowment is 2 units of good x and 3 units of good y, while consumer B's initial endowment is 4 units of good x and 3 units of good y. Neither of them can influence the prices of *x* and *y*. The price ratio p_x/p_y for which equilibrium will take place in pure competition is:

a) 3/2
b) 3/5
c) 2/3
a) 1
b) none of the above

Problem #2

Anne and Tom consume only chips and peanuts (*c* – chips, *p* – peanuts). Regardless of the amounts of the goods consumed, the marginal rate of substitution (MRS_{CP}) for Tom is -2, while for Anne it is -3. Assume that Tom's initial endowment is 3 packages of chips and 3 packages of peanuts, while for Anne it is 6 packages of chips and 10 packages of peanuts. Which of the following statements is true?

a) The described allocation is Pareto efficient.
b) The described allocation is not Pareto efficient because Tom and Anne have differing amounts of each of the goods.
c) The described allocation in not Pareto efficient because Anne could trade 2 packages of peanuts for 1 package of chips and, as a result of this, improve her situation without worsening Tom's situation.
d) The described allocation in not Pareto efficient because Tom could trade 1 package of peanuts for 2 packages of chips and, as a result of this, improve his situation without worsening Anne's situation.
e) None of the above.

Additional problems:

Problem #1

Text as in Problem #2 of the first set above.

vi. $U_1(x,y) = 2x + \ln(y)$, $U_2(x,y) = x + 2\ln(y)$

vii. $U_1(x,y) = 2x^{0.5} + 4y$, $U_2(x,y) = x + y^{0.5}$

viii. $U_1(x,y) = 2\ln(x) + \ln(y)$, $U_2(x,y) = \ln(x) + \ln(y)$

Problem #2

Martina and Phillip are investors who trade stocks of two companies specializing in the sale of agricultural products – Bysto & Co. (x_1) and Polan & Co. (x_2). There are no other investors (besides Martina and Phillip). Revenues from stocks are risky – they depend on whether there was much rain or not during the summer time. Both situations are equally likely. The dividend per single share of Bysto & Co. amounts to 1 zloty if the summer was rainy, or 0 otherwise. For Polan & Co. the contrary holds, i.e. the dividend per single share of that company is 0 if the summer was rainy, or 1 otherwise. Martina owns 100 shares of Bysto &Co. and does not have any shares of Polan & Co., while Phillip has 100 shares of Polan & Co. and does not have any shares of Bysto & Co. Both Martina and Phillip maximize their expected utility given by the following formula: $U_i(x_1,x_2) = \frac{1}{2}\ln x_1 + \frac{1}{2}\ln x_2$.

a) In the Edgeworth box indicate the initial endowment and specify whether it is an efficient allocation (provide a reasoning for your answer).
b) What is the equilibrium price of shares?
c) What can be said about the risk connected with Martina's and Phillip's shares in the initial situation and in equilibrium? Which of these two allocations is less risky?