MICROECONOMICS 3
PROBLEMS #5

PUBLIC GOODS

Problem #1

A certain society consists of 2 groups of consumers. The first one's demand function for a private good (food) is $Q(p) = 10 - 2p$, while for a public good (healthcare) it is $X(p) = 100 - p$. The second group's demand function for the private good is $Q(p) = 8 - 0.8p$ and for the public good it is $X(p) = 200 - p$. Find the graphical expression and algebraic formula of the aggregate demand function for the private and for the public good.

Problem #2

A small town has 2000 inhabitants with identical preferences. There are only 2 goods in this town – the private good and the public good. The utility function of each inhabitant is $U(x_i,y) = x_i + y^{1/2}$, where x_i is the private good of inhabitant i and y is the amount of the public good provided in the town. The cost of the private good is \$1 *per* unit and for the public good it is \$10 *per* unit. Find the Pareto-efficient amount of the public good provided in the town.

Problem #3

A mountain village has 50 inhabitants. As a result of a fire a playroom for children burnt down recently. The village mayor aiming to build a new playroom must make a decision concerning its size. Each inhabitant of the village has the following marginal rate of substitution between square meters of the new playroom and money spent on other goods: $MRS = 1.2 - 0.0004x$, where x is the size (area) of the playroom in m^2. The marginal cost of 1 m^2 of the playroom is 20 PLN. Find the socially efficient size (area) of the new playroom for this village.

Problem #4

All 10 neighbors living in the same street are willing to pay 2 rubles for installing each additional lantern. The neighbors' willingness to pay for this does not depend on the number of already installed lanterns. If the cost of setting up x lanterns is described by the function $C(x) = x^2$, find the Pareto-efficient amount of lanterns installed in the street.

Problem #5 (mercifully deleted)

Problem #6

Public safety is, as its name indicates, a public good. However, new residential districts are often additionally protected by guards. Two types of families live in one of such districts: young couples and old couples. The first of them are not rich and their inverse demand function for guards is given by $P(x) = 10 - 3x$, where x is the number of guards. The inverse demand function of old couples for guards is $P(x) = 20 - x$. The inverse supply of guards is given by $P(x) = 20 + x$.

a) Provide the formula of the aggregate demand for guards function and present it graphically.
b) How many guards should be employed in this district?

Problem #7

Inhabitants of a village use a common grazing field for their cows. The price of a cow is 200. The amount of milk produced by a cow depends on how many cows use the common grazing field. The milk production (in liters) function is $M(C) = 300C - C^2$, where C is the number of cows using this grazing field. The price of 1 liter of milk amounts to 1 and does not depend on the amount of milk supplied.

a) What is the optimal from the village inhabitants' point of view (i.e. maximizing joint profits) number of cows using the grazing field?
b) How many cows will be using it when each farmer is not able to influence the decisions of others regarding using the common grazing field and there exist no regulations as regards the number of cows using this field? Find the difference in joint profits for this situation and the optimum from point a).

Problem #8

Inhabitants of a city consume a private good x (understood as money spent on all private goods a consumer buys so the price of this good is 1) and public good g (using a common ice-rink). The city has 1000 inhabitants and each of them has the following utility function: $U(x,g) = x - 100/g$, where x is the value of private consumption and g – the size of the public ice-rink in m^2. The cost of the ice-rink is 10 per m^2. Each inhabitant has identical income amounting to 1000.

a) Find the Pareto-efficient ice-rink size.
b) Assume each inhabitant will bear an equal share in the costs of the ice-rink (i.e. $10g/1000$). The ice-rink size is decided in a poll, where each inhabitant has 1 vote. What will be the ice-rink size decided in this way?

Problem #9

A public good costs 99 zloty. Three persons vote on whether to provide this good or not. Their reservation prices are $r_1 = 90$, $r_2 = 30$, and $r_3 = 30$, respectively. In case of a positive result of the vote, which is in fact a majority vote, each of these persons bears 1/3 of the total costs of this good's provision. What will be the result of the vote?