MICROECONOMICS 3
PROBLEMS #6

GENERAL EQUILIBRIUM. WELFARE

Problem #1

A society consists of two individuals (*A* and *B*), and the utility possibility frontier is given by $U_A + 2U_B = 200$. Plot the utility frontier. For each point below, determine values of U_A and U_B that maximize the social welfare (W). Show the solutions both algebraically and graphically.

(a) Assume a "Nietzschean social welfare function": $W(U_A; U_B) = \max\{U_A; U_B\}$.

(b) Use a Rawlsian criterion: $W(U_A; U_B) = \min\{U_A; U_B\}$.

(c) Suppose social welfare is given by $W(U_A; U_B) = U_A^{1/2} U_B^{1/2}$.

Problem #2

A parent has two children, named A and B, and she loves both of them equally. She has a total of $1,000 to give to them. Assuming different utility functions of the parent (in the points below), determine how she will divide the money between her children in each case.

(a) $U(a; b) = \log(a) + \log(b)$

(b) $U(a; b) = \min\{a,b\}$

(c) $U(a; b) = \max\{a,b\}$

(d) $U(a; b) = a^2 + b^2$

(e) $U(a; b) = a^{0.5} + b^{0.5}$

(f) $U(a; b) = -1/a - 1/b$

Problem #3

Romeo and Juliet consume only one good, spaghetti. Romeo likes spaghetti, but he also likes Juliet to be happy and he knows that spaghetti makes her happy. Juliet likes spaghetti, but she also likes Romeo to be happy and she knows that spaghetti makes Romeo happy. Romeo's utility function is $U_R(S_R; S_J) = S_R^a S_J^{1-a}$ and Juliet's utility function is $U_J(S_J; S_R) = S_J^a S_R^{1-a}$, where S_J and S_R are the amount of spaghetti for Romeo and the amount of spaghetti for Juliet, respectively. There is a total of 24 units of spaghetti to be divided between them.

(a) Suppose that $a = 2/3$. If Romeo got to allocate the 24 units of spaghetti exactly as he wanted to, how much would he give himself, and how much would he give Juliet?

(b) Again assuming $a = 2/3$, if Juliet got to allocate the spaghetti exactly as she wanted to, how much would she take for herself, and how much would she give Romeo?

(c) What are the Pareto optimal allocations when $a = 2/3$? (Hint: An allocation will not be Pareto optimal if both persons' utility will be increased by a gift from one to the other.)

(d) Suppose that $a = 1/3$. If Romeo got to allocate the spaghetti, how much would he give himself? If Juliet got to allocate the spaghetti, how much would she give herself?

(e) When $a = 1/3$, at the Pareto optimal allocations what do Romeo and Juliet disagree about?

Problem #4

One possible method of determining a social preference relation is rank-order voting (also known as the *Borda count*). Each voter is asked to rank all of the alternatives, and the voters' scores for each alternative are then added over all voters. The total score for an alternative is called its Borda count. For any two alternatives, x and y, if the Borda count of x is smaller than the Borda count for y, then x is socially preferred to y.

Suppose that there are a finite number of alternatives to choose from and that every individual has complete, reflexive, and transitive preferences. For the time being, let us also suppose that individuals are never indifferent between any two different alternatives but always prefer one to the other.

(a) Is the social preference ordering defined in this way complete, reflexive, and transitive?

(b) If everyone prefers x to y, will the Borda count rank x as socially preferred to y? Explain your answer.

(c) Suppose that there are two voters (1 and 2) and three candidates (x, y, and z). Suppose that Voter 1 ranks the candidates, x first, z second, and y third. Suppose that Voter 2 ranks the candidates, y first, x second, and z third. What is the Borda count for each of the three candidates?

(d) Now suppose that it is discovered that candidate z once lifted a beagle by the ears. Voter 1, who has rather large ears himself, is appalled and changes his ranking to x first, y second, z third. Voter 2, who picks up his own children by the ears, is favorably impressed and changes his ranking to y first, z second, x third. Now what is the Borda count for each of the three candidates?

(e) Does the social preference relation defined by the Borda count have the property that social preferences between x and y depend only on how people rank x versus y and not on how they rank other alternatives? Explain (you can use the example in point d).

Problem #5

Hatfield and McCoy hate each other but love corn whiskey. Because they hate for each other to be happy, each wants the other to have less whiskey. Hatfield's utility function is $U_H(W_H;W_M) = W_H - W_M^2$ and McCoy's utility function is $U_M(W_M;W_H) = W_M - W_H^2$, where W_M is McCoy's daily whiskey consumption and W_H is Hatfield's daily whiskey consumption (both measured in quarts). There are 4 quarts of whiskey to be allocated.

(a) If McCoy got to allocate all of the whiskey, how would he allocate it?

(b) If Hatfield got to allocate all of the whiskey, how would he allocate it?

(c) If each of them gets 2 quarts of whiskey, what will the utility of each of them be? If a bear spilled 2 quarts of their whiskey and they divided the remaining 2 quarts equally between them, what would the utility of each of them be? If it is possible to throw away some of the whiskey, is it Pareto optimal for them each to consume 2 quarts of whiskey?

(d) If it is possible to throw away some whiskey and they must consume equal amounts of whiskey, how much should they throw away?