MICROECONOMICS 3
PROBLEMS #7

PUBLIC GOODS

Problem #1
A certain society consists of 2 groups of consumers. The first one's demand function for a private good (food) is $Q(p) = 10 – 2p$, while for a public good (healthcare) it is $X(p) = 100 – p$. The second group's demand function for the private good is $Q(p) = 8 – 0.8p$ and for the public good it is $X(p) = 200 – p$. Find the graphical expression and algebraic formula of the aggregate demand function for the private and for the public good.

Problem #2
A small town has 2000 inhabitants with identical preferences. There are only 2 goods in this town – the private good and the public good. The utility function of each inhabitant is $U(x_i,y) = x_i + y^{1/2}$, where x_i is the private good of inhabitant i and y is the amount of the public good provided in the town. The cost of the private good is \$1 per unit and for the public good it is \$10 per unit. Find the Pareto-efficient amount of the public good provided in the town.

Problem #3
A mountain village has 50 inhabitants. As a result of a fire a playroom for children burnt down recently. The village mayor aiming to build a new playroom must make a decision concerning its size. Each inhabitant of the village has the following marginal rate of substitution between square meters of the new playroom and money spent on other goods: MRS = $1.2 – 0.0004x$, where x is the size (area) of the playroom in m^2. The marginal cost of 1 m^2 of the playroom is 20 PLN. Find the socially efficient size (area) of the new playroom for this village.

Problem #4
All 10 neighbors living in the same street are willing to pay 2 rubles for installing each additional lantern. The neighbors' willingness to pay for this does not depend on the number of already installed lanterns. If the cost of setting up x lanterns is described by the function $C(x) = x^2$, find the Pareto-efficient amount of lanterns installed in the street.

Problem #5
Public safety is, as its name indicates, a public good. However, new residential districts are often additionally protected by guards. Two types of families live in one of such districts: young couples and old couples. The first of them are not rich and their inverse demand function for guards is given by $P(x) = 10 – 3x$, where x is the number of guards. The inverse demand function of old couples for guards is $P(x) = 20 – x$. The inverse supply of guards is given by $P(x) = 20 + x$.

a) Provide the formula of the aggregate demand for guards function and present it graphically.
b) How many guards should be employed in this district?

Problem #6
Inhabitants of a village use a common grazing field for their cows. The price of a cow is 200. The amount of milk produced by a cow depends on how many cows use the common grazing field. The milk production (in liters) function is $M(C) = 300C – C^2$, where C is the number of

cows using this grazing field. The price of 1 liter of milk amounts to 1 and does not depend on the amount of milk supplied.

a) What is the optimal from the village inhabitants' point of view (i.e. maximizing joint profits) number of cows using the grazing field?
b) How many cows will be using it when each farmer is not able to influence the decisions of others regarding using the common grazing field and there exist no regulations as regards the number of cows using this field? Find the difference in joint profits for this situation and the optimum from point a).

Problem #7

Inhabitants of a city consume a private good x (understood as money spent on all private goods a consumer buys so the price of this good is 1) and public good g (using a common ice-rink). The city has 1000 inhabitants and each of them has the following utility function: $U(x,g) = x - 100/g$, where x is the value of private consumption and g – the size of the public ice-rink in m^2. The cost of the ice-rink is 10 per m^2. Each inhabitant has identical income amounting to 1000.

a) Find the Pareto-efficient ice-rink size.
b) Assume each inhabitant will bear an equal share in the costs of the ice-rink (i.e. $10g/1000$). The ice-rink size is decided in a poll, where each inhabitant has 1 vote. What will be the ice-rink size decided in this way?

Problem #8

A public good costs 99 zloty. Three persons vote on whether to provide this good or not. Their reservation prices are $r_1 = 90$, $r_2 = 30$, and $r_3 = 30$, respectively. In case of a positive result of the vote, which is in fact a majority vote, each of these persons bears 1/3 of the total costs of this good's provision. What will be the result of the vote?

Problem #9

There is a river used by two mines and two laundries. The mines are located upriver, and they use the river to dump untreated sewage. Sewage treatment would generate additional cost for the mines. Laundries operate downriver, and the river water pollution increases their cost of work. Both laundries and mines are lobbing for and against, respectively, a government intervention aimed at reducing the river water pollution. They declare the net values as presented in the table, and the Vickrey-Clarke-Groves (VCG) mechanism is introduced. Who and in what amount will pay the VCG tax?

	Government intervention	No government intervention
Mine A		30
Mine B		38
Laundry C	40	
Laundry D	30	

Problem #10

A public good is being considered to be delivered to a society consisting of three individuals: A, B and C. Individual i, where $i = \{A,B,C\}$, assigns value v_i to having the good delivered, and needs to pay cost c_i for the good provision.

Individual	c_i	v_i
A	100	45
B	100	45
C	100	250

a) Suppose the decision about the good provision (or not) will be taken based on a majority yes-no vote, in which each individual will be asked whether she is willing to pay cost ci for having the good delivered. What will be the result of the vote? Is the outcome Pareto efficient?
b) Now suppose individuals declare their values and this will determine whether the good is provided or not. The Vickrey-Clarke-Groves (VCG) mechanism is introduced. Who and in what amount will pay the tax? Is the outcome Pareto efficient?
c) In b, are the individuals incentivised to reveal their values truthfully? Answer using the example of individual A. Assume that B and C declare their true values, and A might modify it if she has an incentive to do so. Will she modify the value and declare it untruthfully? What value would A need to declare in order for the good not to be delivered? Would it be better for A than revealing her value truthfully?

<u>Problem #11</u>

In a city called Economics, there live two inhabitants: Ann and Bob. The city council has agreed for providing public good *G*, which would be financed only from the inhabitants' contributions. Ann and Bob have identical utility functions: $U(X, G) = 2ln(X) + ln(G)$, where *X* denotes private consumption and *G* the consumption of the public good. The amount of the public good provided is equal to the number of units purchased by Ann and Bob together. Both Ann and Bob has income of 200, each. The unit price of each good is equal to 1.

a) How many units of *G* will be delivered without any intervention in the market mechanism?
b) What amount of *G* is socially optimal?
c) Suppose the city council is unhappy with the market allocation and levied a lump-sum tax of 10 on each inhabitant. Revenues from the tax are spent for providing the public good. Is the Pareto efficient allocation reach then?
d) Suppose now that the city council is still unhappy with the allocation and, instead of the lump-sum tax from c, the council charges Ann with the tax of 50 and Bob with the tax of 25. The tax revenues are again used for the provision of the public good. How many units of *G* are finally delivered? How many units of *G* will Ann buy, and how many will Bob buy? Compare this situation with point c).