

Lecture #5

General equilibrium Welfare

The presentation is based on the slides by Hal R. Varian, *Intermediate Microeconomics*.

Social Choice

- ◆ Usually, many Pareto efficient allocations exist.
- ◆ Different economic states will be preferred by different individuals.
- ◆ How can individual preferences be “aggregated” into a social preference over all possible economic states?

Rational Preference Relation

- ◆ A preference relation is called **rational** if the preference order is both transitive and complete.
- ◆ **Complete**: Any two different bundles can be compared. $\forall (x, y) \in \mathbf{X}$, either $x \succeq y$, $y \succeq x$, or both.
- ◆ **Transitive**: A “consistency” requirement, enabling a ranking. If a consumer thinks that X is at least as good as Y and that Y is at least as good as Z, then the consumer thinks that X is at least as good as Z.
For all choices x, y , and z , if $x \succeq y \wedge y \succeq z$ then $x \succeq z$

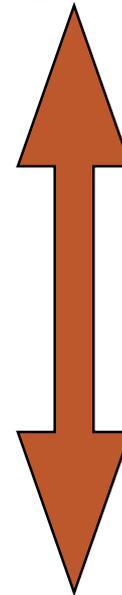
Aggregating Preferences

- ◆ x, y, z denote different economic states.
- ◆ 3 agents: Bill, Bertha and Bob.
- ◆ Use simple majority voting to decide a state?

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

More preferred



Less preferred

Bill, Bertha and Bob have rational preferences.

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y

y beats z

z beats x

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y

y beats z

z beats x

**No socially
best alternative!**

Majority voting does not always aggregate transitive individual preferences into a transitive social preference.

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Will ranking work?

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Will ranking work?

Rank-Order Vote Results
(low score wins)

x-score = 6

y-score = 6

z-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Will ranking work?

Rank-Order Vote Results
(low score wins)

x-score = 6

y-score = 6

z-score = 6

No state is selected!

Rank-order voting is indecisive in this case.

Manipulating Preferences

- ◆ Most voting schemes are manipulable.
- ◆ The outcome of majority voting can depend on the order in which pairs of variants are voted.
- ◆ The outcome of rank-order voting can be influenced by introducing new choice options.

Manipulating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y (1)

y beats z (2)

z beats x (3)

For example:

- ❖ If the voting concerns only x and y, and y and z (1 and 2), then x will win.
- ❖ If the voting concerns only x and y, and z and x (1 and 3), then z will win.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α (3)	y(3)
α (4)	x(4)	α (4)

Rank-Order Vote

These are truthful preferences.

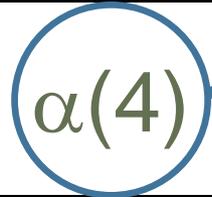
Bob introduces a new alternative (α).

Manipulating Preferences

Bill	Bertha	Bob
$x(1)$	$y(1)$	$z(1)$
$y(2)$	$z(2)$	$x(2)$
$z(3)$	$\alpha(3)$	$y(3)$
$\alpha(4)$	$x(4)$	$\alpha(4)$

Rank-Order Vote

Bob introduces a new alternative (α) and, then, lies about his preference order.



Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	$\alpha(2)$
z(3)	$\alpha(3)$	x(3)
$\alpha(4)$	x(4)	y(4)

Rank-Order Vote

Bob introduces a new alternative (α) and, then, lies about his preference order.

x-score = 8

y-score = 7

z-score = 6

α -score = 9

z wins!

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α (3)	y(3)
α (4)	x(4)	α (4)

Rank-Order Vote

Bob introduces a new alternative (α) and, then, lies about his preference order.

If he didn't lie:

x-score = 7

y-score = 6

z-score = 6

α -score = 11

Desirable Voting Rule Properties

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y , then so should the voting rule.
3. Social preference between x and y should depend on individuals' preferences between x and y only.

Desirable Voting Rule Properties

- ◆ **Kenneth Arrow's Impossibility Theorem:**
The only voting rule with all of properties 1, 2 and 3 is dictatorial.
- ◆ (Dictatorship means a social outcome determined by a single individual.)
- ◆ Implication is that a non-dictatorial voting rule requires giving up at least one of properties 1, 2 or 3.

Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y , then so should the voting rule.
3. Social preference between x and y should depend on individuals' preferences between x and y only.

Give up which one of these in order to build a social welfare function?

Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y , then so should the voting rule.
3. ~~Social preference between x and y should depend on individuals' preferences between x and y only.~~

Give up which one of these in order to build a social welfare function?

Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y , then so should the voting rule.
- ~~3. Social preference between x and y should depend on individuals' preferences between x and y only.~~

There is a variety of voting procedures with both properties 1 and 2.

Social Welfare Functions

◆ $u_i(x)$ is individual i 's utility from overall allocation x .

◆ Utilitarian:
$$W = \sum_{i=1}^n u_i(x).$$

◆ Weighted-sum:
$$W = \sum_{i=1}^n a_i u_i(x)$$
 with each $a_i > 0$.

◆ Minimax:
(Rawlsian)
$$W = \min\{u_1(x), \dots, u_n(x)\}.$$

Social Welfare Function

- ◆ Suppose social welfare depends only on individuals' own allocations, instead of overall allocations.
- ◆ That is, utility of individual i is $u_i(x_i)$, rather than $u_i(x)$.
- ◆ Then, the social welfare function is some function of individuals' utility functions:

$$W = f(u_1(x_1), \dots, u_n(x_n))$$

where f is an increasing function.

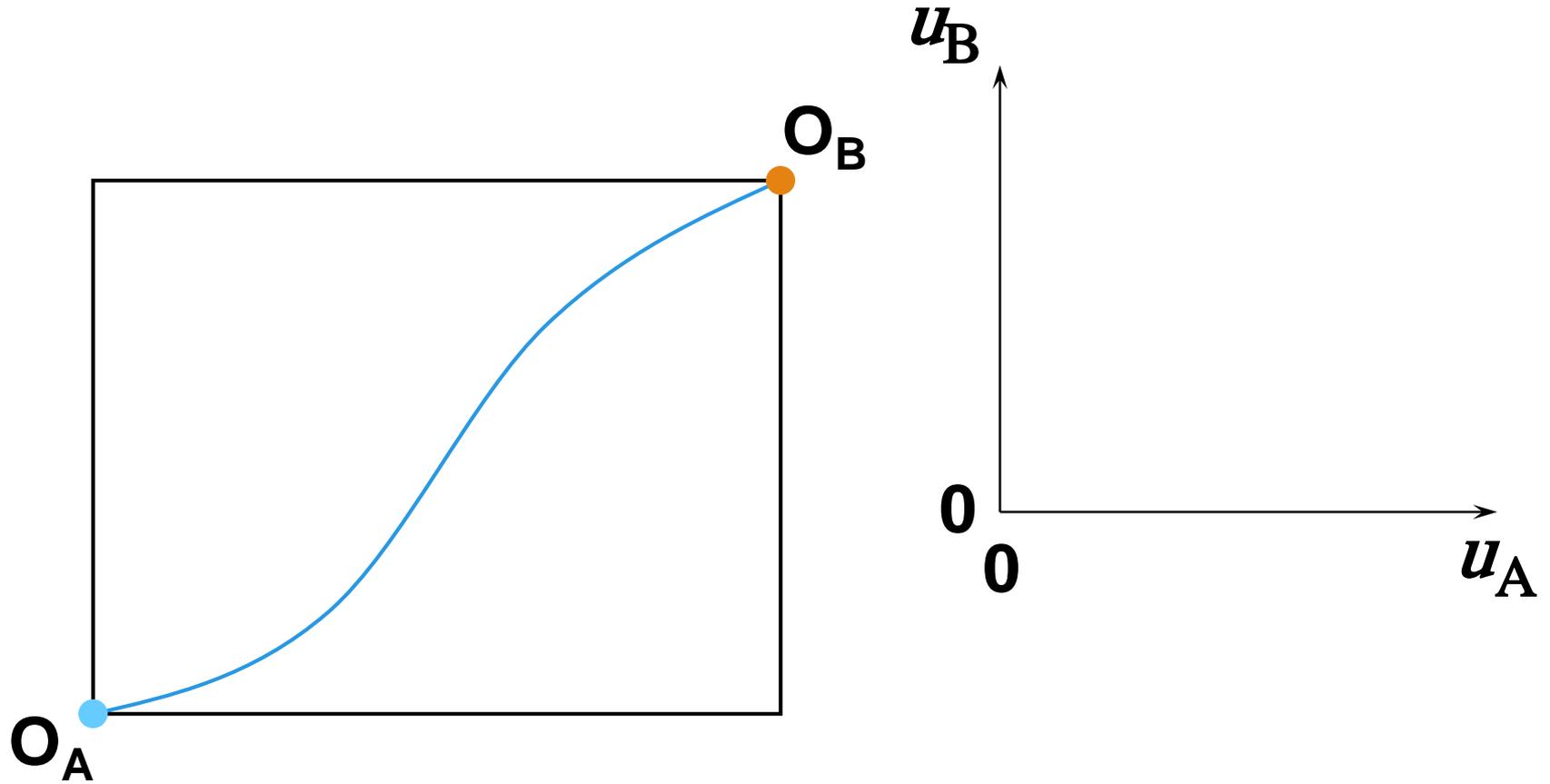
Social Optima & Efficiency

- ◆ Any social optimal allocation must be Pareto efficient.
- ◆ Why?

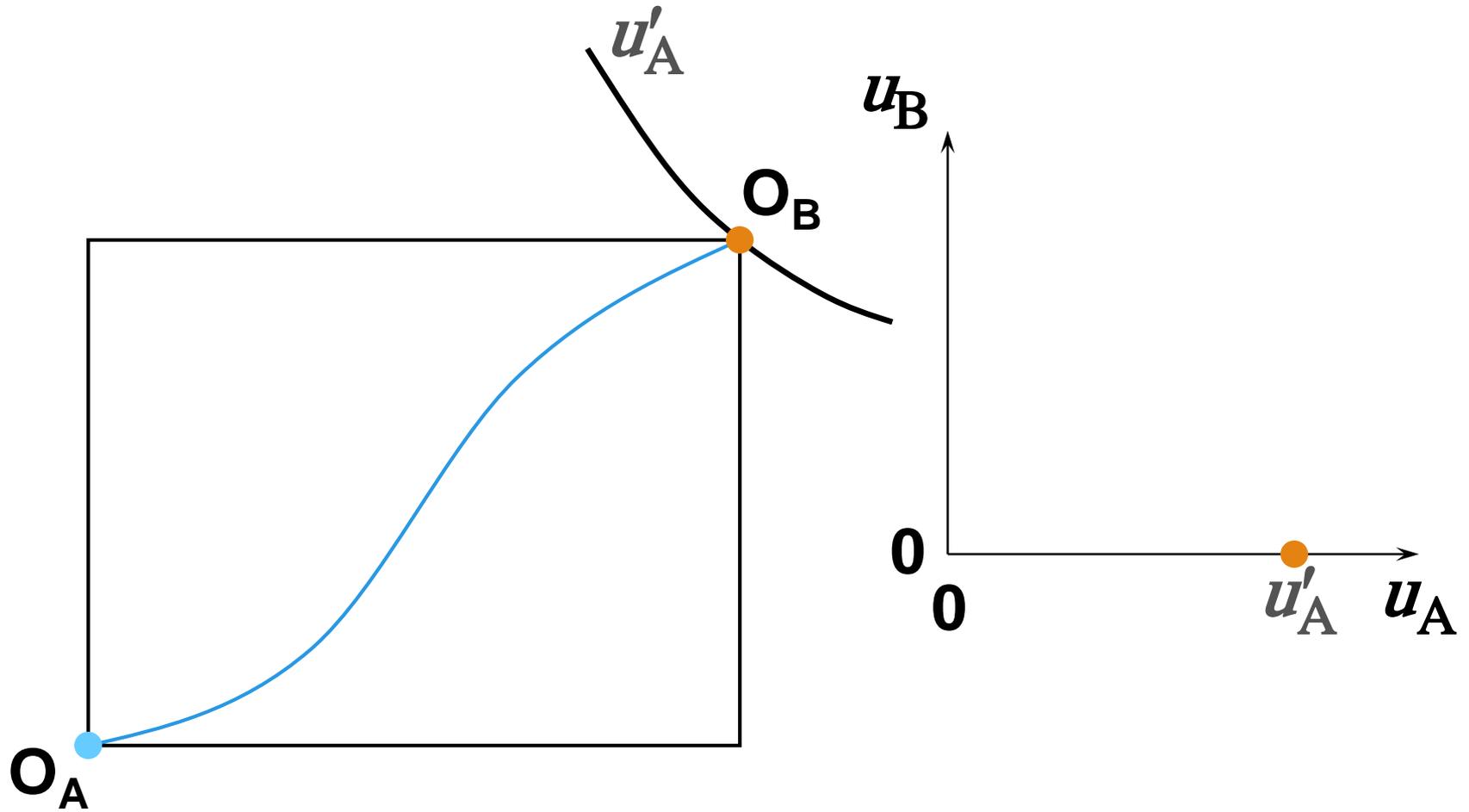
Social Optima & Efficiency

- ◆ Any social optimal allocation must be Pareto efficient.
- ◆ Why?
- ◆ If not, then somebody's utility can be increased without reducing anyone else's utility.
- ◆ That is, social suboptimality \Rightarrow inefficiency.

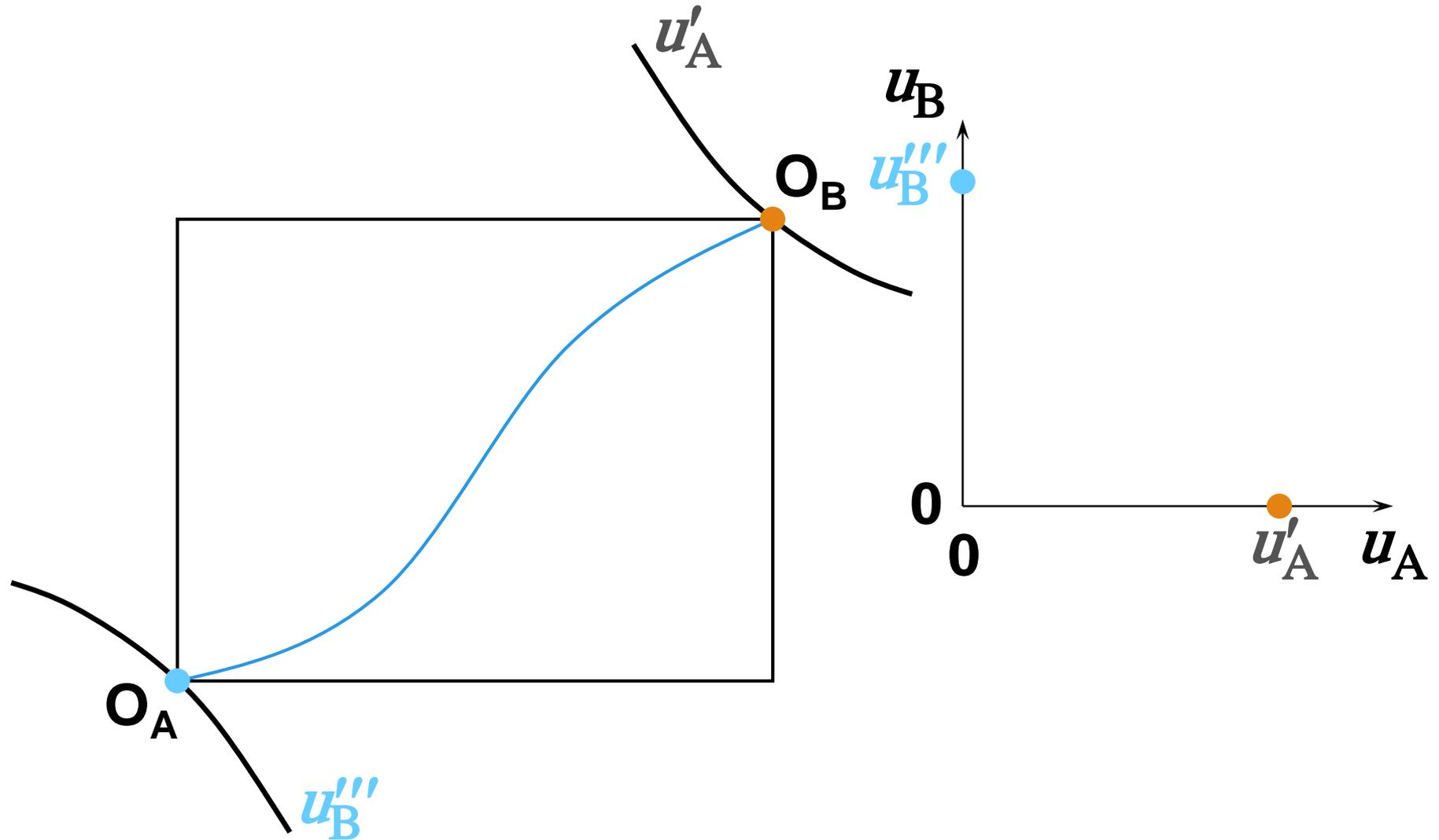
Utility Possibilities



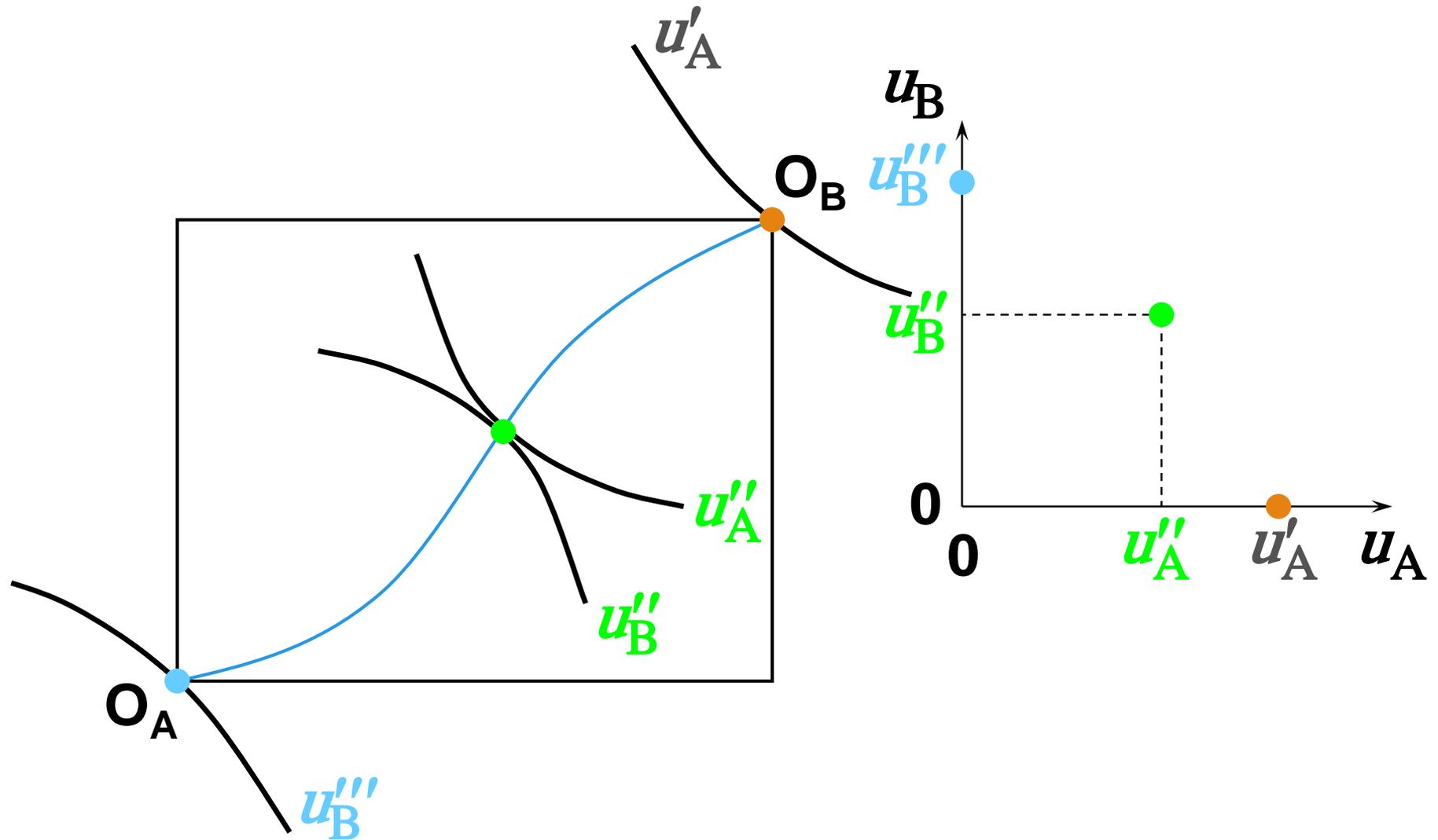
Utility Possibilities



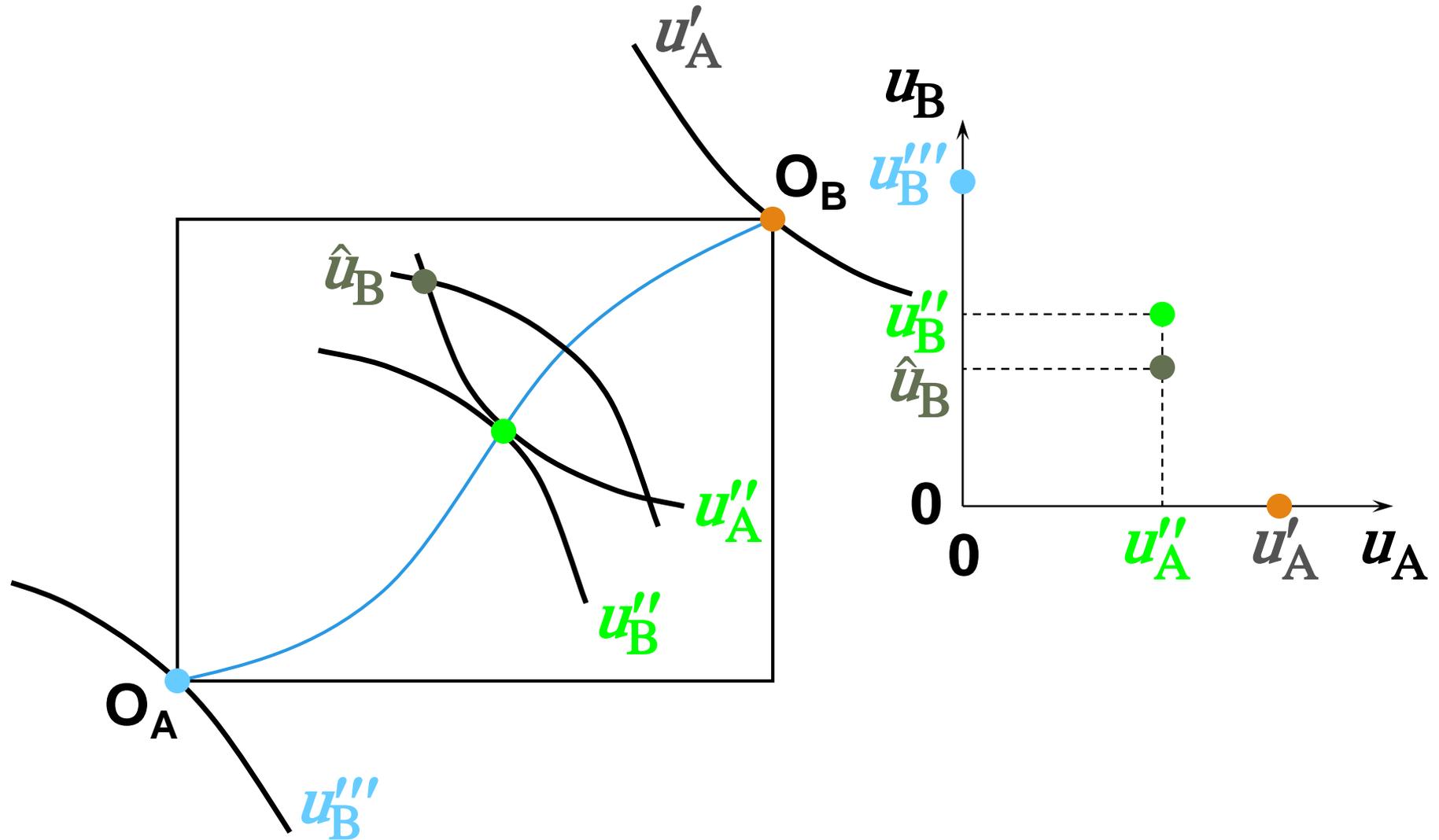
Utility Possibilities



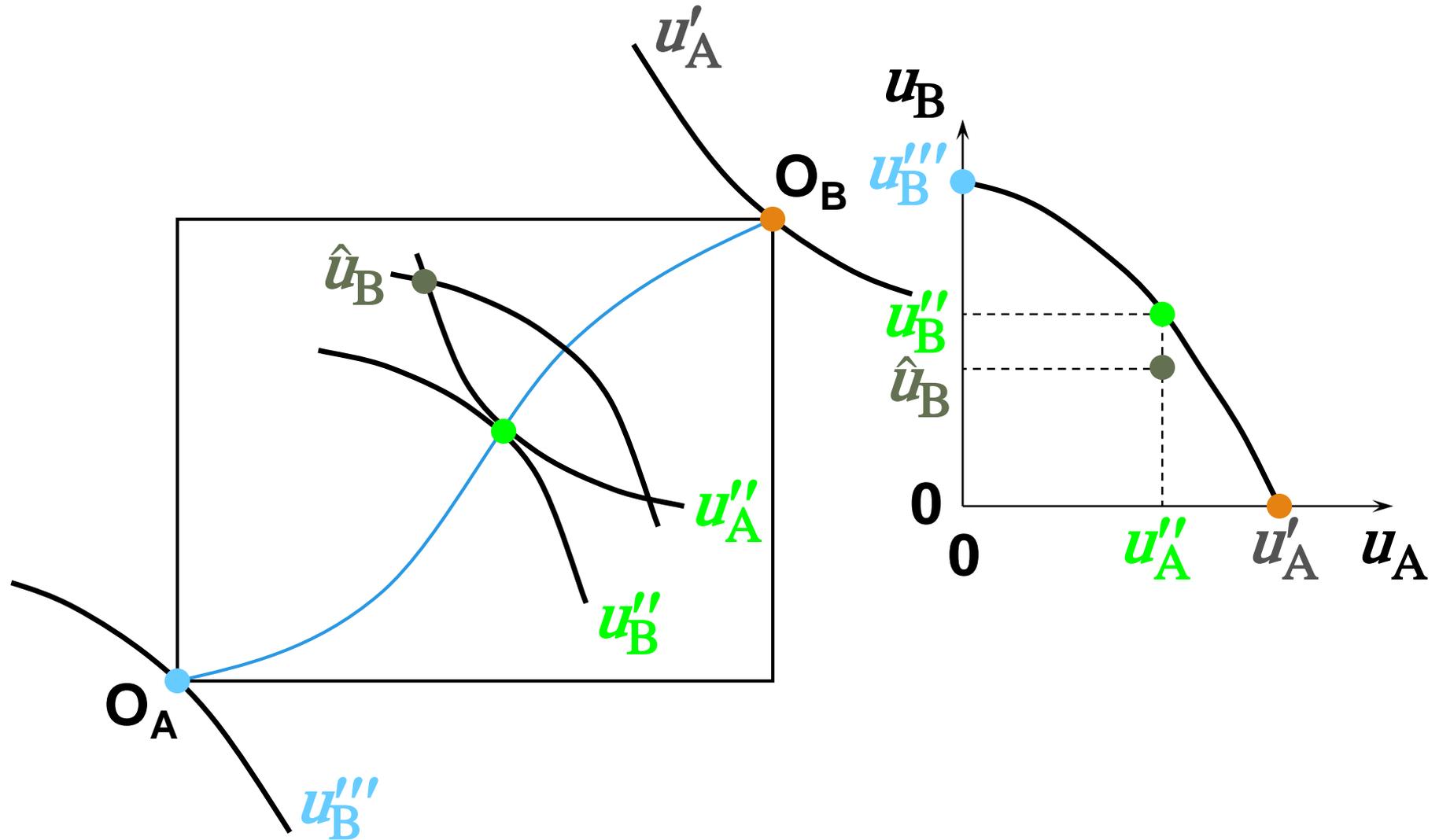
Utility Possibilities



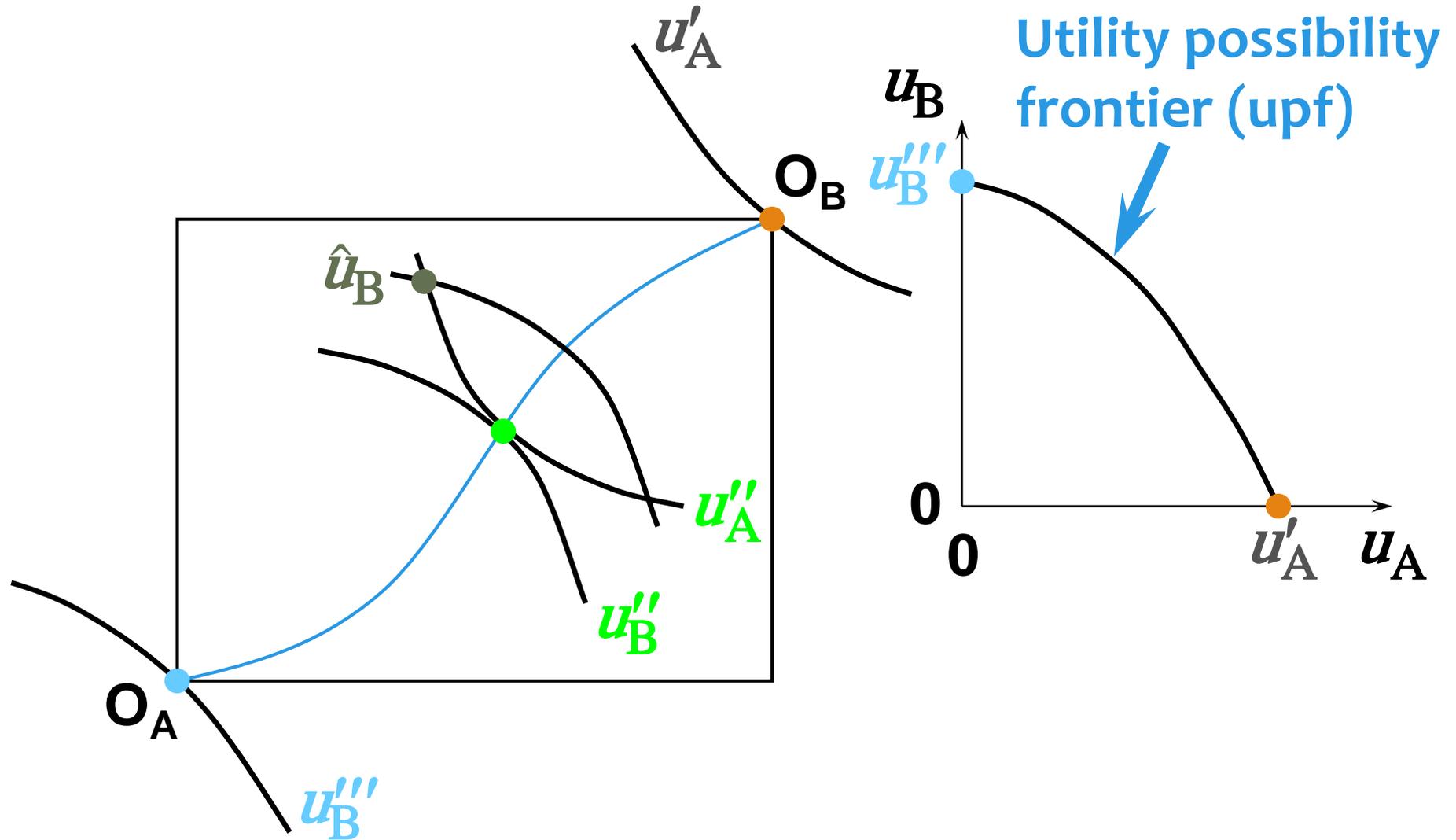
Utility Possibilities



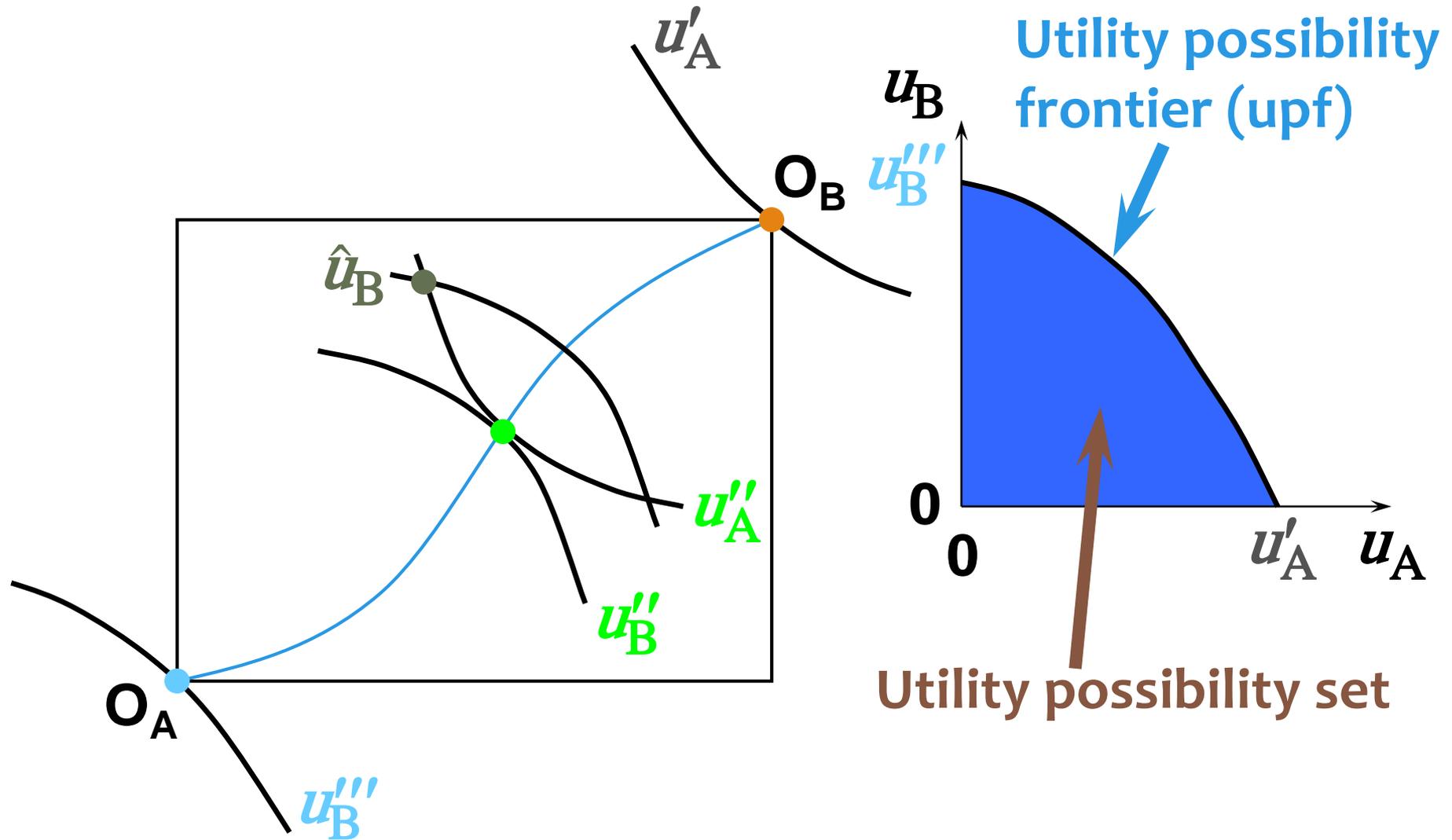
Utility Possibilities



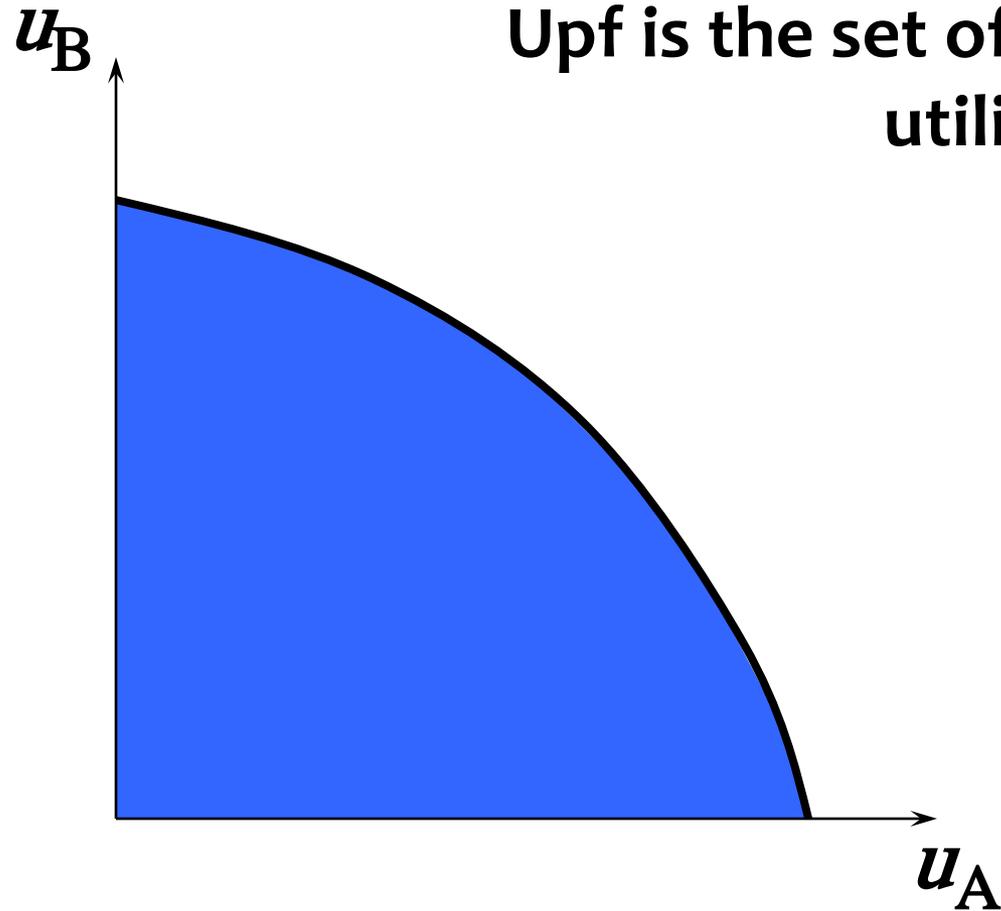
Utility Possibilities



Utility Possibilities

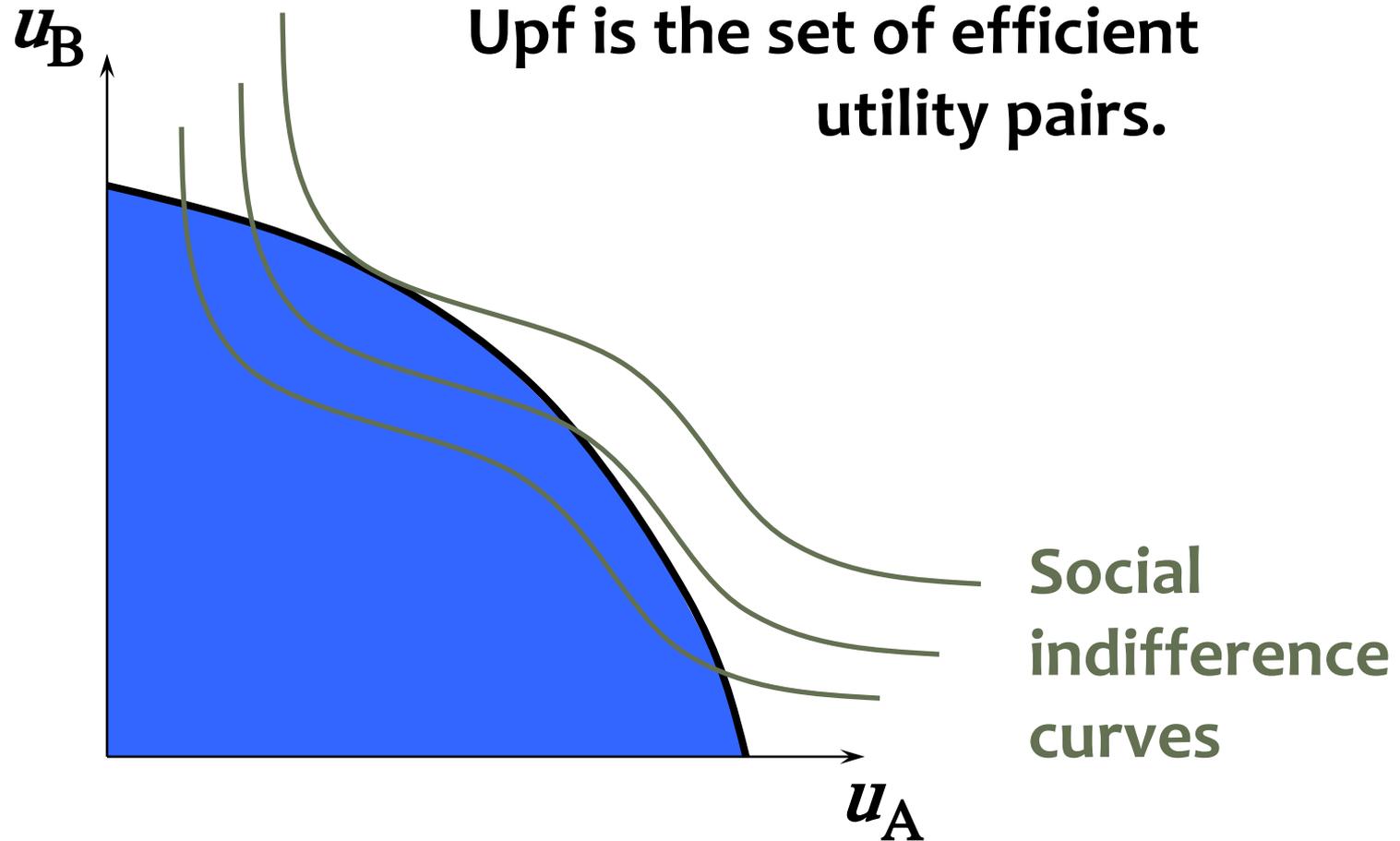


Social Optima & Efficiency

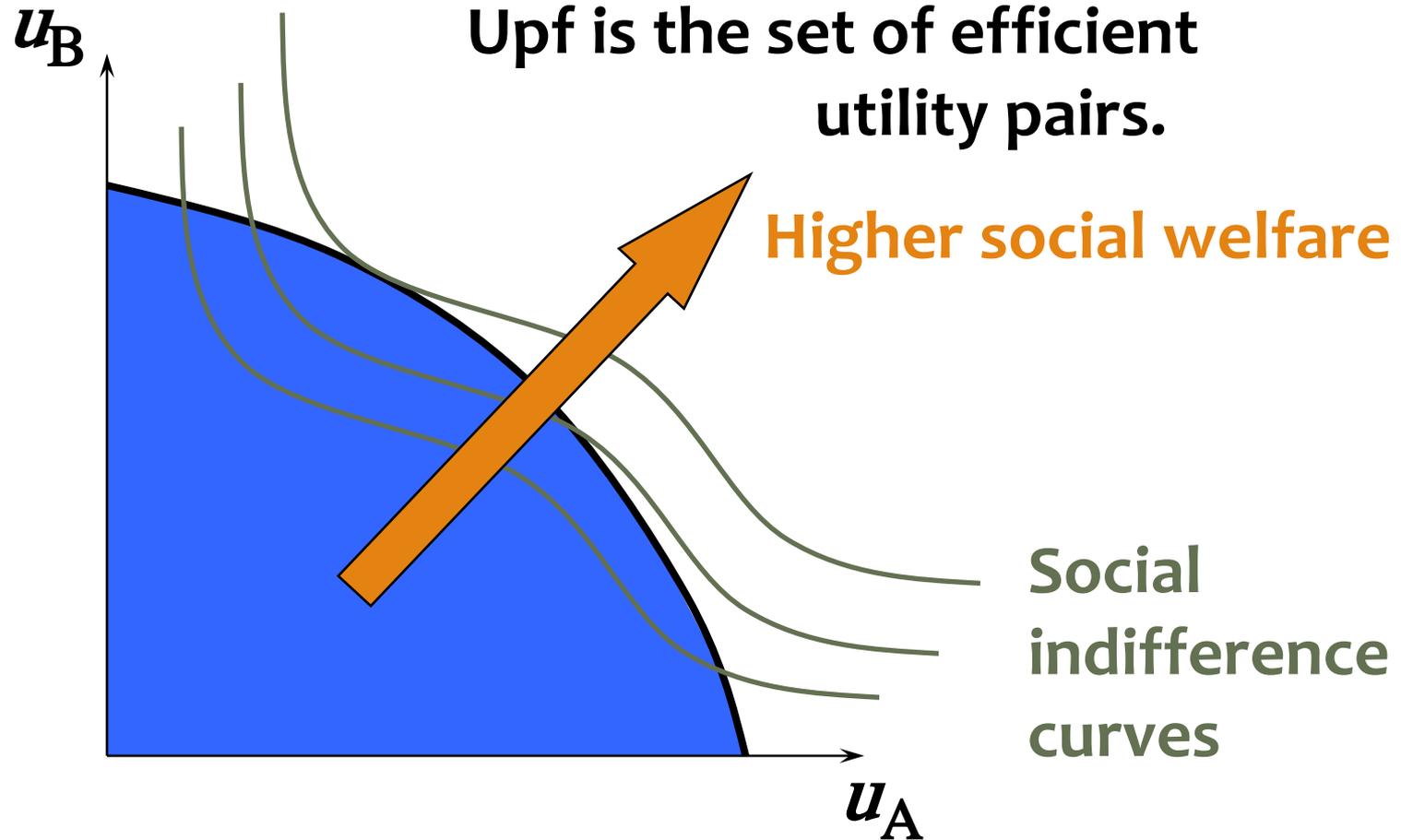


Upf is the set of efficient utility pairs.

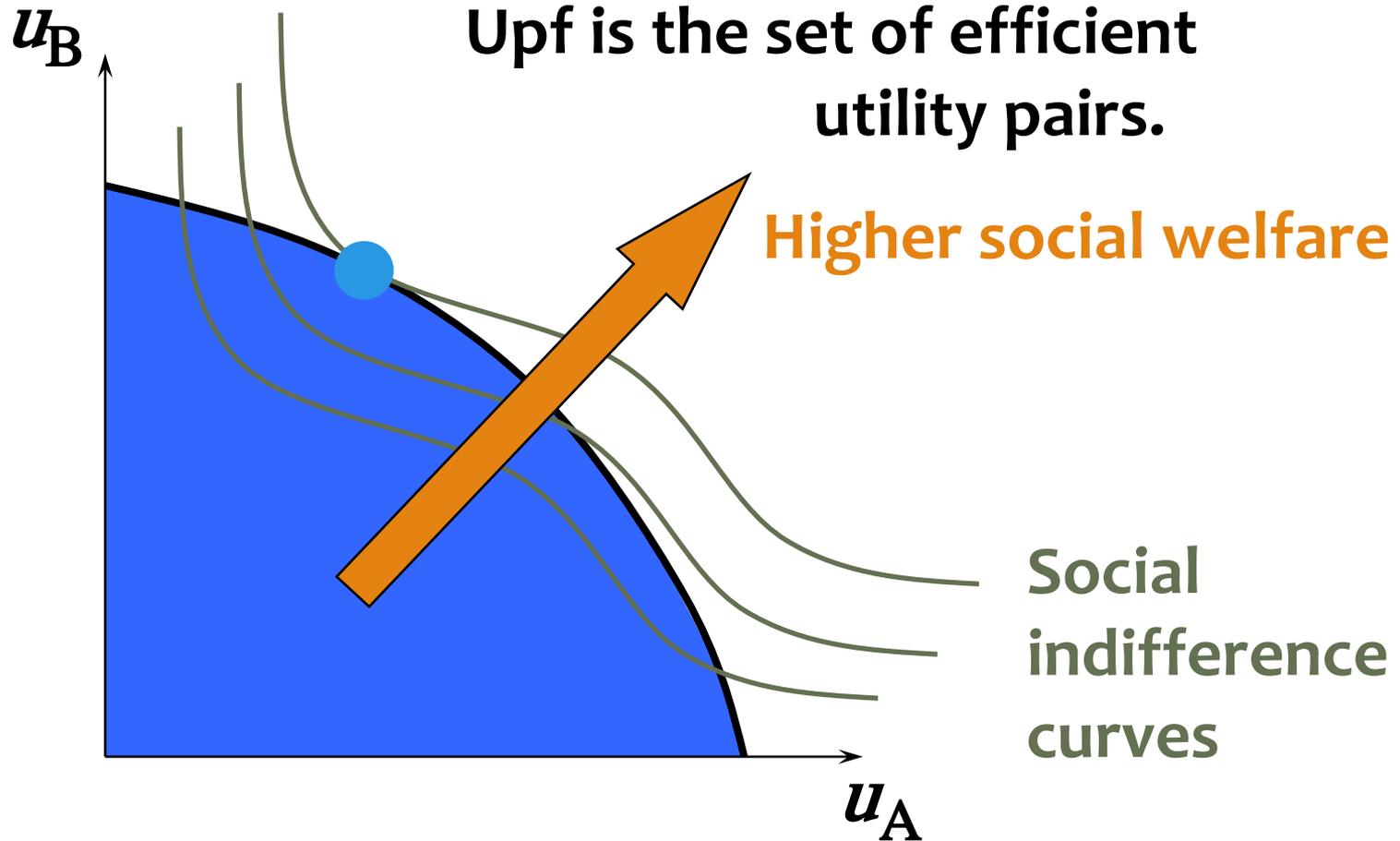
Social Optima & Efficiency



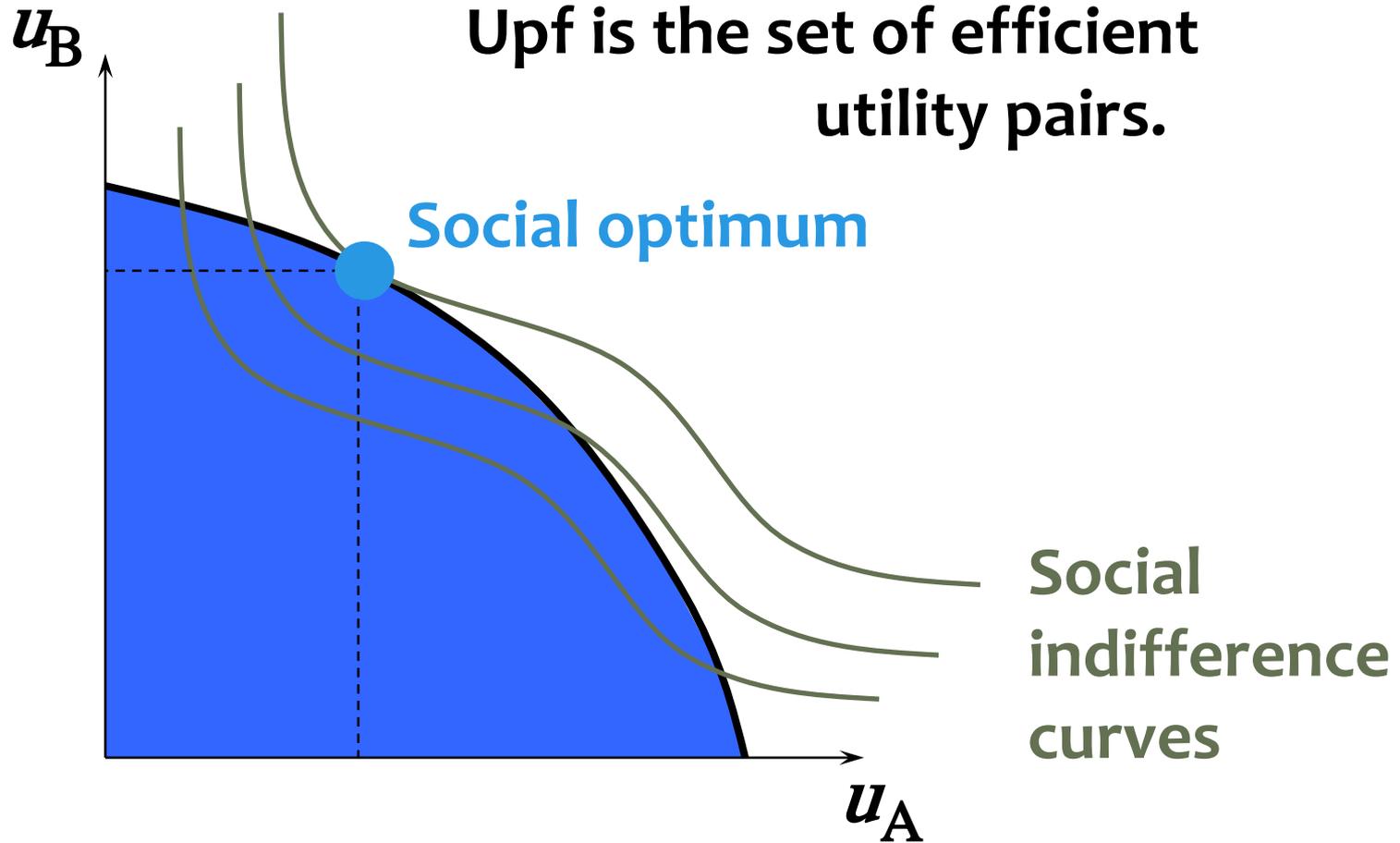
Social Optima & Efficiency



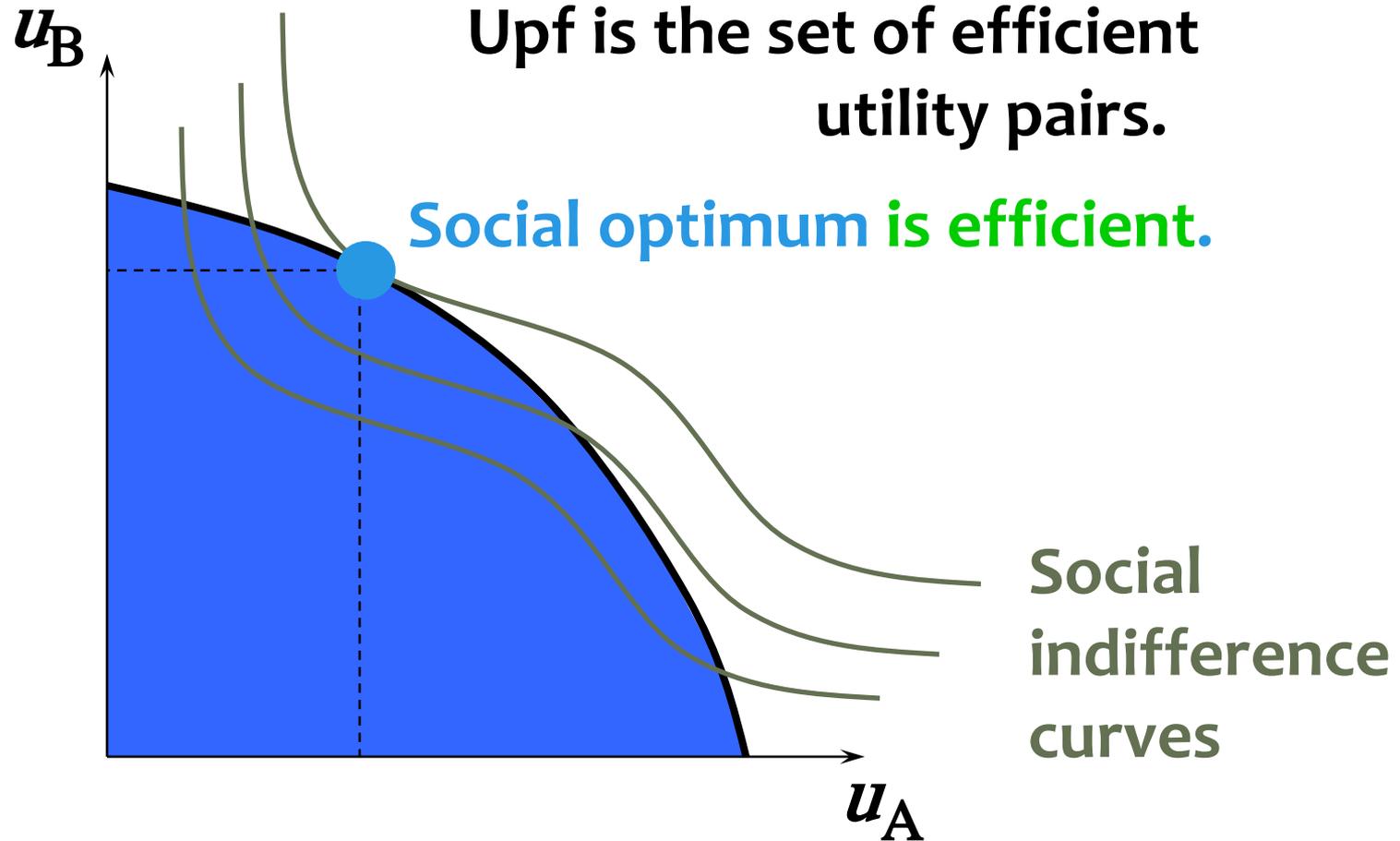
Social Optima & Efficiency



Social Optima & Efficiency



Social Optima & Efficiency



Fair Allocations

- ◆ Some Pareto efficient allocations are “unfair”.
- ◆ E.g., one consumer eats everything is efficient, but “unfair”.
- ◆ Can competitive markets guarantee that a “fair” allocation can be achieved?
- ◆ If agent A prefers agent B’s allocation to his own, then agent A envies agent B.
- ◆ An allocation is **fair** if it is Pareto efficient and envy free (equitable).

Fair Allocations

- ◆ 2 agents, same endowments.
- ◆ Now trade is conducted in competitive markets.
- ◆ Must the post-trade allocation be fair?

Fair Allocations

- ◆ 2 agents – A and B.
- ◆ The same endowments – goods 1 and 2.
- ◆ Now trade is conducted in competitive markets.
- ◆ Must the post-trade allocation be fair?
- ◆ Yes. Why?

Fair Allocations

- ◆ Endowment of each agent is (ω_1, ω_2) .
- ◆ Post-trade bundles are (x_1^A, x_2^A) and (x_1^B, x_2^B) .
- ◆ Then $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1 + p_2 \omega_2$
and $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1 + p_2 \omega_2$.

Fair Allocations

- ◆ Suppose agent A envies agent B.
- ◆ This means that $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- ◆ But as (x_1^A, x_2^A) is the best bundle A can afford, this implies (x_1^B, x_2^B) is not affordable for A:

$$p_1 x_1^B + p_2 x_2^B > p_1 \omega_1 + p_2 \omega_2.$$

- ◆ Contradiction: Both agents started with equal endowments, so this would mean that agent B cannot afford the bundle (x_1^B, x_2^B) either.

Fair Allocations

- ◆ Suppose agent A envies agent B.
- ◆ This means that $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- ◆ But as (x_1^A, x_2^A) is the best bundle A can afford, this implies (x_1^B, x_2^B) is not affordable for A:

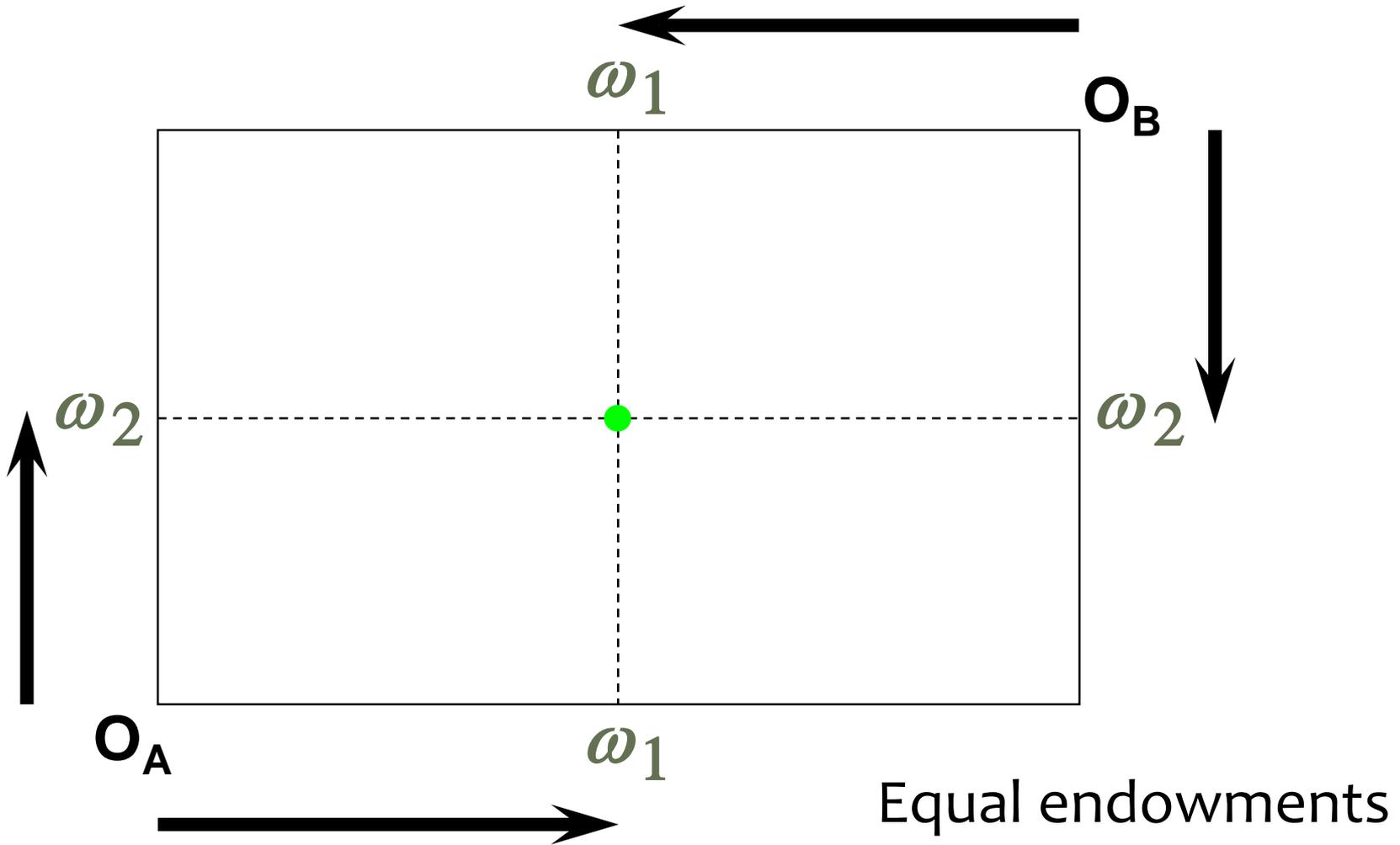
$$p_1 x_1^B + p_2 x_2^B > p_1 \omega_1 + p_2 \omega_2.$$

- ◆ Contradiction: Both agents started with equal endowments, so this would mean that agent B cannot afford the bundle (x_1^B, x_2^B) either.
- ◆ Conclusion: **It is impossible for agent A to envy agent B in these circumstances.**

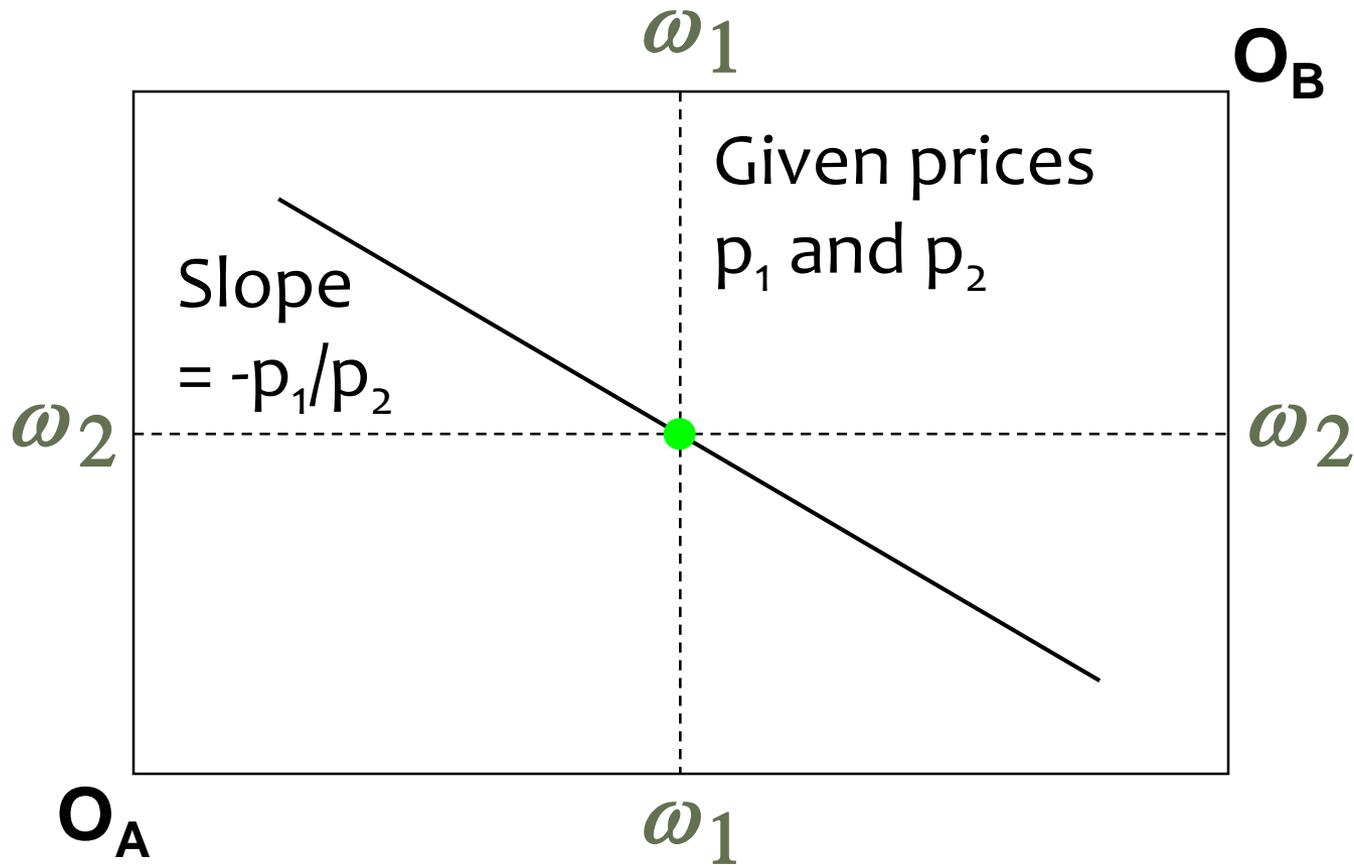
Fair Allocations

- ◆ Suppose agent A envies agent B.
- ◆ This means that $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- ◆ But as (x_1^A, x_2^A) is the best bundle A can afford, this implies $(x_1^B, x_2^B) \succ_B (x_1^A, x_2^A)$.
If every agent's endowment is identical, then trading in competitive markets results in a fair allocation.
- ◆ Competitive markets started with equal endowments, so this would mean that agent B cannot afford the bundle (x_1^B, x_2^B) either.
- ◆ Conclusion: **It is impossible for agent A to envy agent B in these circumstances.**

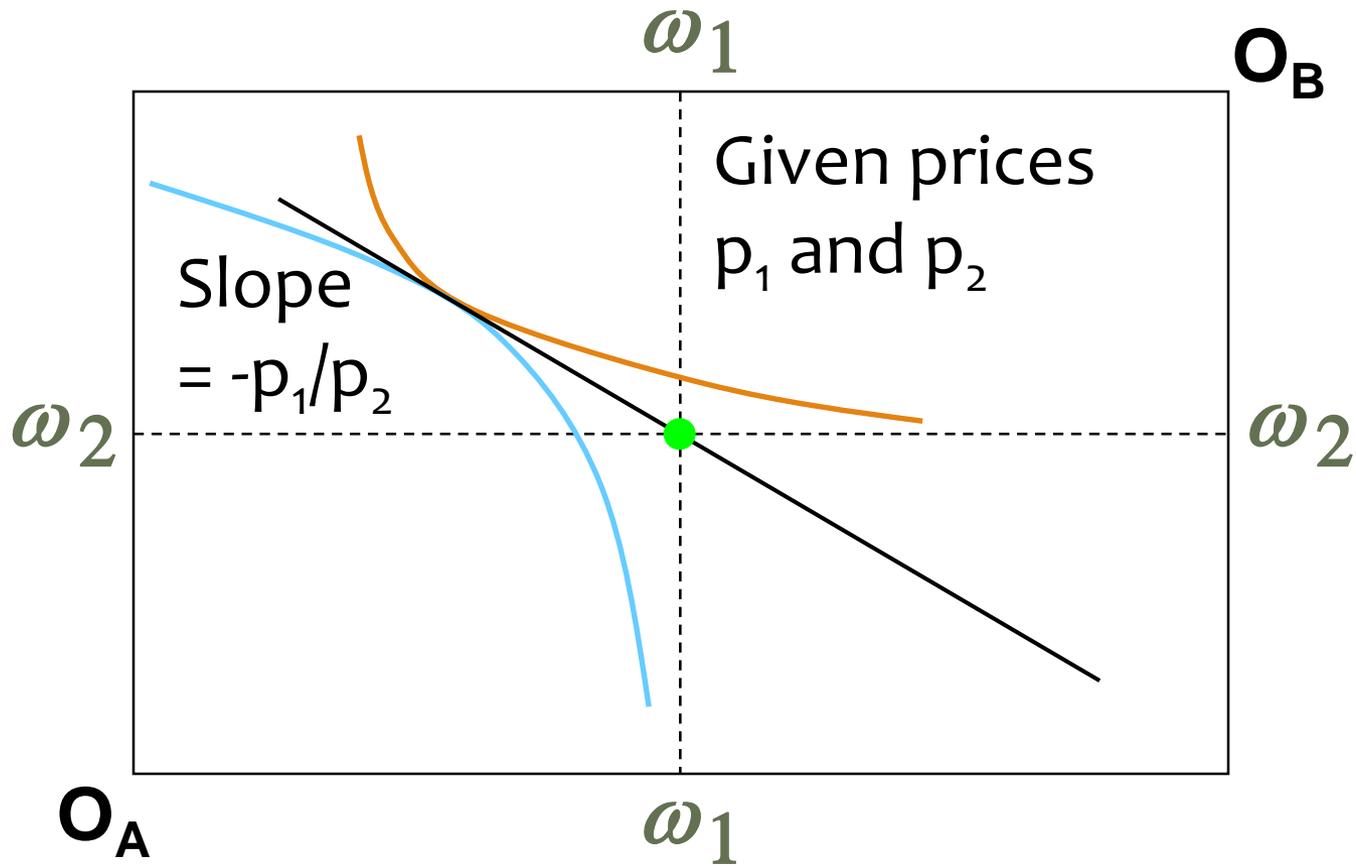
Fair Allocations



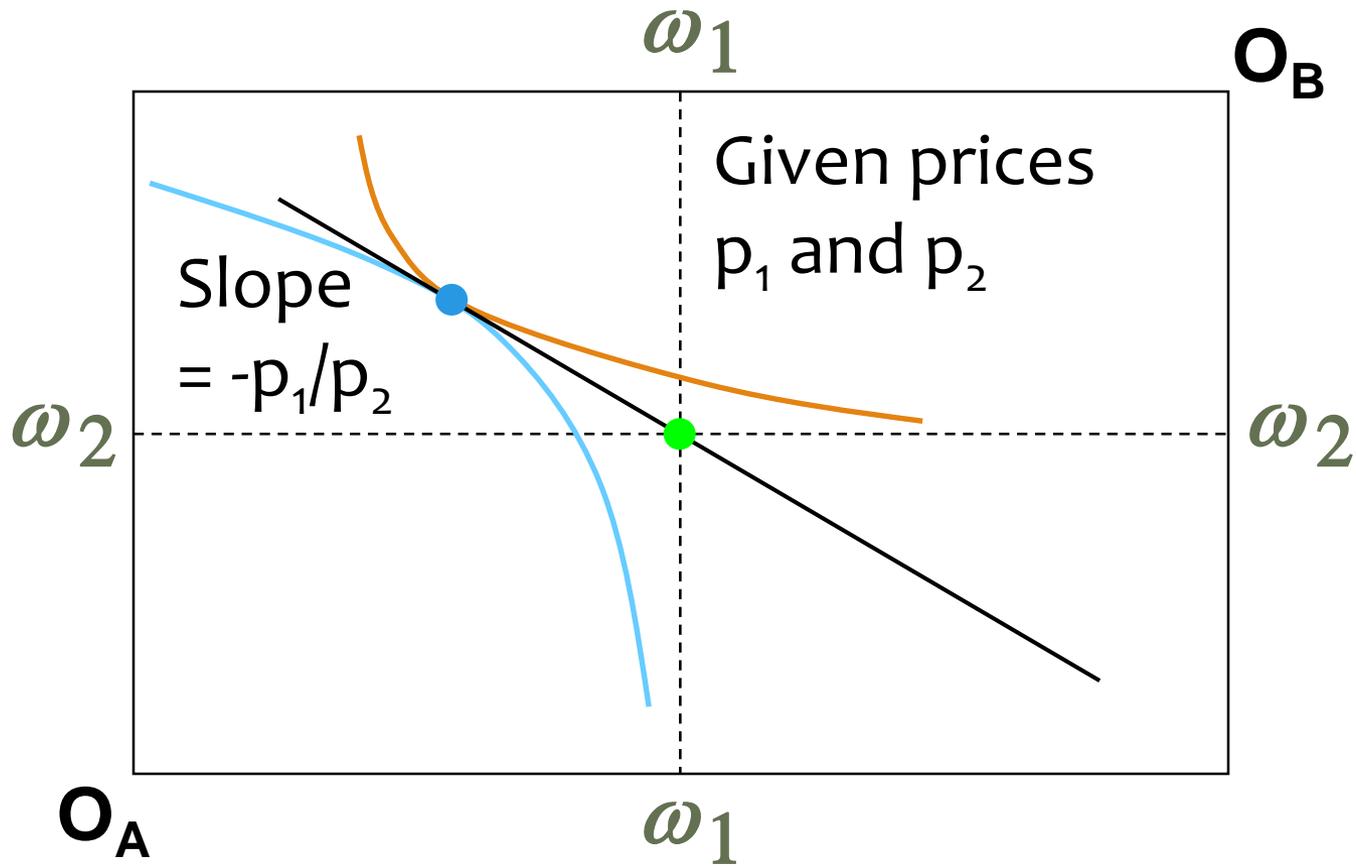
Fair Allocations



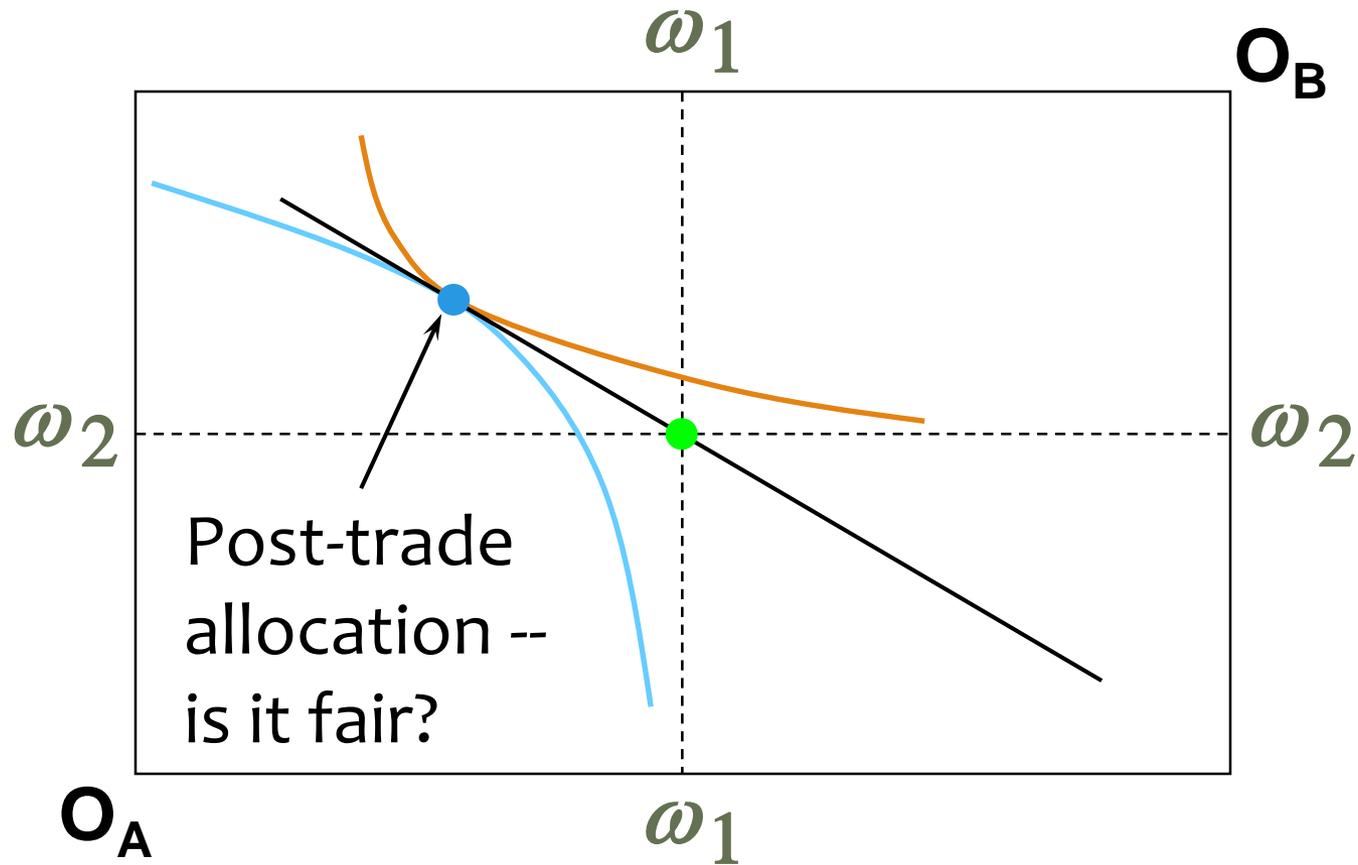
Fair Allocations



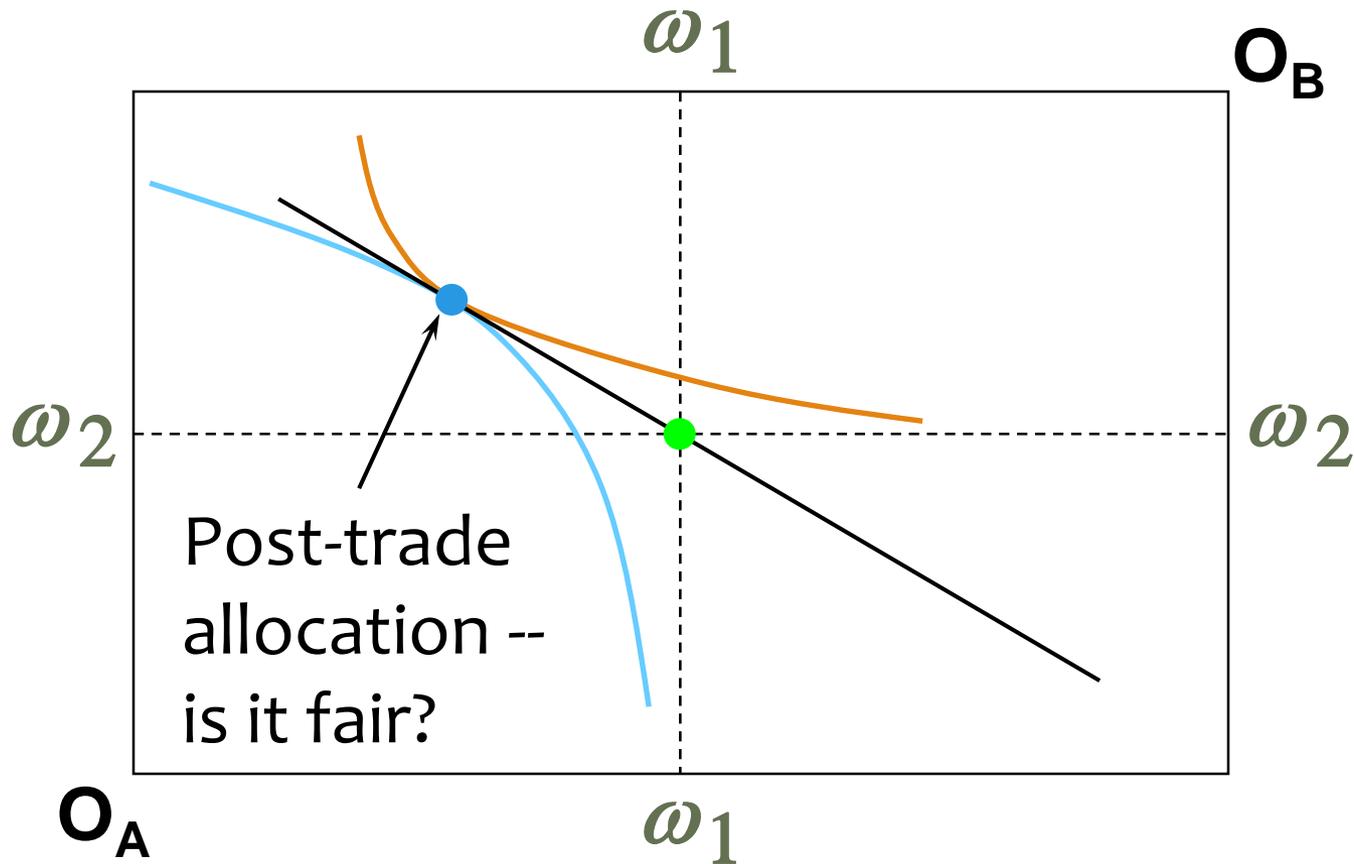
Fair Allocations



Fair Allocations

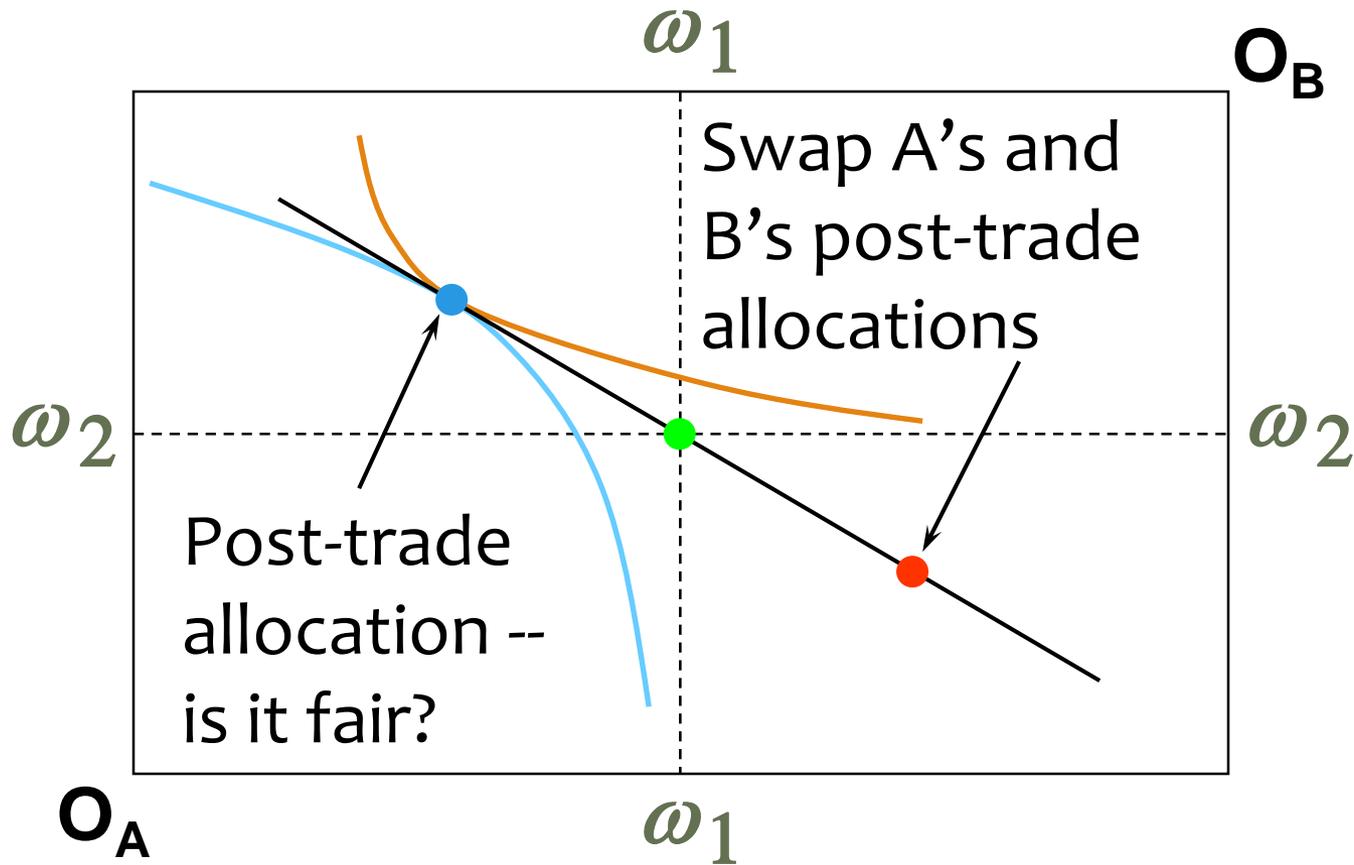


Fair Allocations



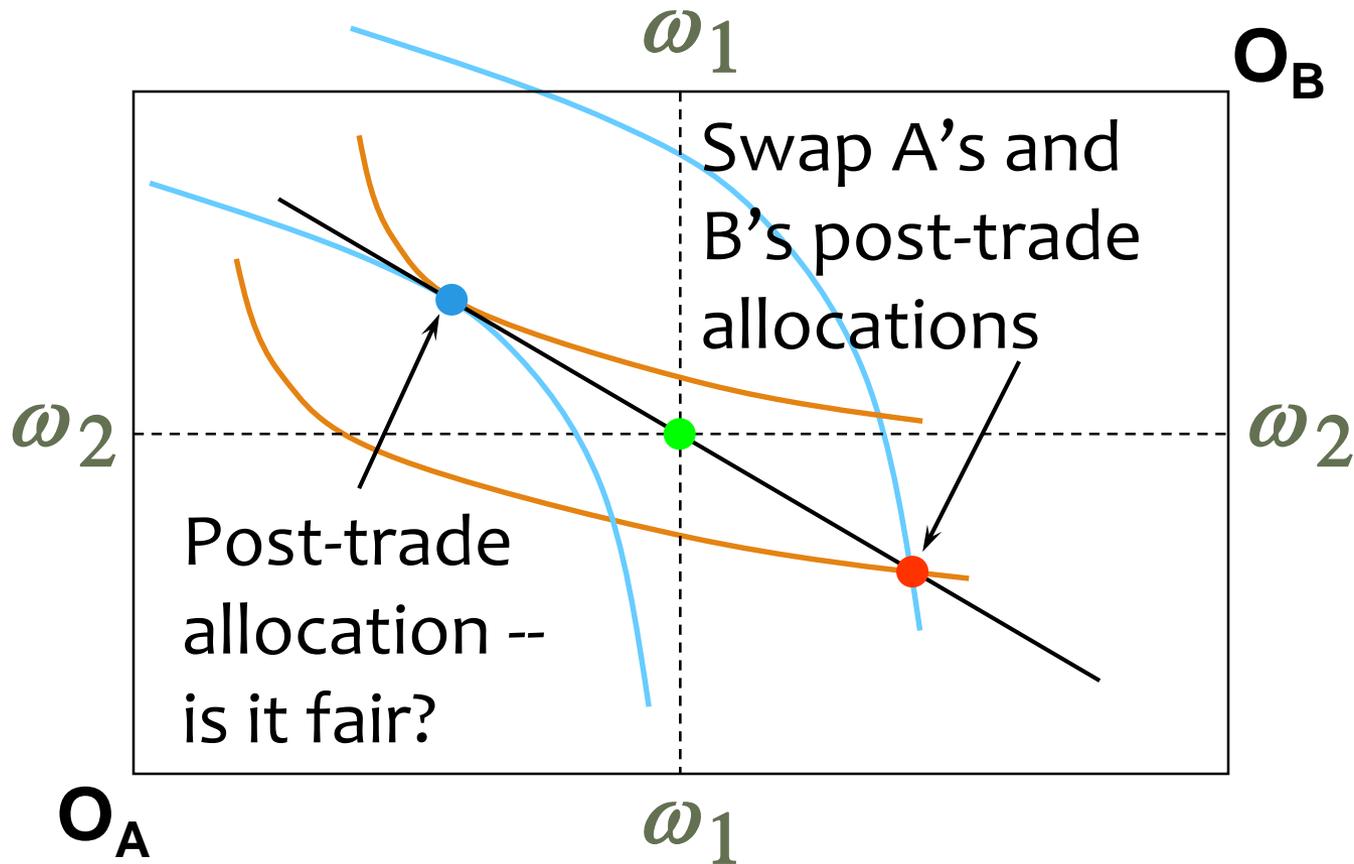
It is Pareto efficient, but does any of the agents envy?

Fair Allocations



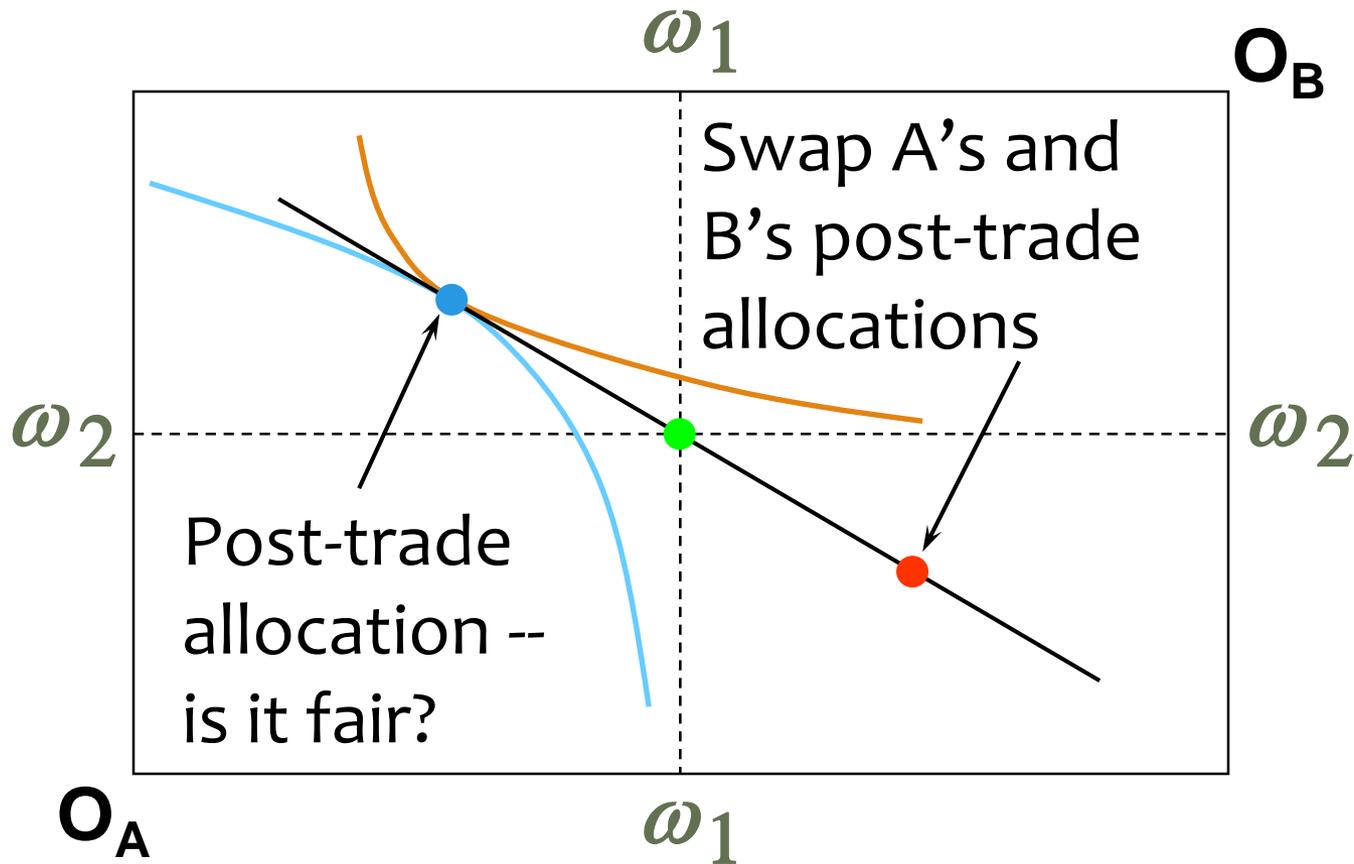
It is Pareto efficient, but does any of the agents envy?

Fair Allocations



Each person prefers the post-trade allocation to the swapped allocation.

Fair Allocations



Post-trade allocation is Pareto-efficient and envy-free; hence it is fair.