an agent is the set of all nodes of the tree that cannot be differentiated by the agent. For example, the simultaneous-move game depicted at the beginning of this section can be represented by the game tree in Figure 15.2. In this figure, the shaded area indicates that Column cannot differentiate which of these decisions Row made at the time when Column must make his own decision. Hence, it is just as if the choices are made simultaneously.

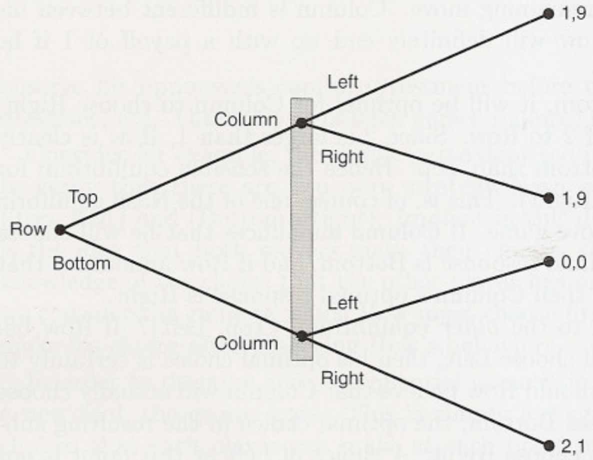

Figure 15.2 **Information set.** This is the extensive form to the original simultaneous-move game. The shaded information set indicates that Column is not aware of which choice Row made when he makes his own choice.

Thus the extensive form of a game can be used to model everything in the strategic form *plus* information about the sequence of choices and information sets. In this sense the extensive form is a more powerful concept than the strategic form, since it contains more detailed information about the strategic interactions of the agents. It is the presence of this additional information that helps to eliminate some of the Nash equilibria as "unreasonable."

EXAMPLE: A simple bargaining model

Two players, A and B, have \$1 to divide between them. They agree to spend at most three days negotiating over the division. The first day, A will make an offer, B either accepts or comes back with a counteroffer the next day, and on the third day A gets to make one final offer. If they cannot reach an agreement in three days, both players get zero.

A and B differ in their degree of impatience: A discounts payoffs in the future at a rate of α per day, and B discounts payoffs at a rate of β per day. Finally, we assume that if a player is indifferent between two offers, he will accept the one that is most preferred by his opponent. This idea is that the opponent could offer some arbitrarily small amount that would make the player strictly prefer one choice, and that this assumption allows us to approximate such an "arbitrarily small amount" by zero. It turns out that there is a unique subgame perfect equilibrium of this bargaining game.[3]

As suggested above, we start our analysis at the end of the game, right before the last day. At this point A can make a take-it-or-leave-it offer to B. Clearly, the optimal thing for A to do at this point is to offer B the smallest possible amount that he would accept, which, by assumption, is zero. So if the game actually lasts three days, A would get 1 and B would get zero (i.e., an arbitrarily small amount).

Now go back to the previous move, when B gets to propose a division. At this point B should realize that A can guarantee himself 1 on the next move by simply rejecting B's offer. A dollar next period is worth α to A this period, so any offer less than α would be sure to be rejected. B certainly prefers $1 - \alpha$ now to zero next period, so he should rationally offer α to A, which A will then accept. So if the game ends on the second move, A gets α and B gets $1 - \alpha$.

Now move to the first day. At this point A gets to make the offer and he realizes that B can get $1 - \alpha$ if he simply waits until the second day. Hence A must offer a payoff that has at least this present value to B in order to avoid delay. Thus he offers $\beta(1 - \alpha)$ to B. B finds this (just) acceptable and the game ends. The final outcome is that the game ends on the first move with A receiving $1 - \beta(1 - \alpha)$ and B receiving $\beta(1 - \alpha)$.

Figure 15.3A illustrates this process for the case where $\alpha = \beta < 1$. The outermost diagonal line shows the possible payoff patterns on the first day, namely all payoffs of the form $x_A + x_B = 1$. The next diagonal line moving towards the origin shows the present value of the payoffs if the game ends in the second period: $x_A + x_B = \alpha$. The diagonal line closest to the origin shows the present value of the payoffs if the game ends in the third period; this equation for this line is $x_A + x_B = \alpha^2$. The right angled path depicts the minimum acceptable divisions each period, leading up to the final subgame perfect equilibrium. Figure 15.3B shows how the same process looks with more stages in the negotiation.

It is natural to let the horizon go to infinity and ask what happens in the infinite game. It turns out that the subgame perfect equilibrium division

[3] This is a simplified version of the Rubinstein-Ståhl bargaining model; see the references at the end of the chapter for more detailed information.