

PUBLIC GOODS

Problem #1

A certain society consists of only 2 consumers. The first one's demand function for a private good is $Q(p) = 10 - 2p$, while for a public good it is $X(p) = 100 - p$. The second consumer's demand function for the private good is $Q(p) = 8 - 0.8p$ and for the public good it is $X(p) = 200 - p$. Find the graphical expression and algebraic formula of the aggregate demand function for the private good and for the public good.

Problem #2

A small town has 2000 inhabitants with identical preferences. There are only 2 goods in this town – the private good and the public good. The utility function of each inhabitant is $U(x_i, y) = x_i + y^{1/2}$, where x_i is the private good of inhabitant i and y is the amount of the public good provided in the town. The cost of the private good is \$1 *per* unit and for the public good it is \$10 *per* unit. Find the Pareto-efficient amount of the public good provided in the town.

Problem #3

A mountain village has 50 inhabitants. As a result of a fire a playroom for children burnt down recently. The village mayor aiming to build a new playroom must make a decision concerning its size. Each inhabitant of the village has the following marginal rate of substitution between square meters of the new playroom and money spent on other goods: $MRS = 1.2 - 0.0004x$, where x is the size (area) of the playroom in m^2 . The marginal cost of 1 m^2 of the playroom is 20 PLN. Find the socially efficient size (area) of the new playroom for this village.

Problem #4

A married couple possessing a large villa has unambiguously defined preferences regarding the use of paper tissues and cupboards for clothes. Both spouses have the same utility function $U(t, c) = t + 2000c^{1/2}$, where t is the amount of tissue packages and c is the amount of cupboards. The tissues are considered a private good for the couple and the cupboards are the common good. A package of tissues costs 1 PLN and the cupboard for clothes – 1000 PLN. Each of the spouses earns at least 10 000 PLN per year. Find the Pareto-efficient amount of cupboards for clothes in this couple's villa.

Problem #5

All 10 neighbors living in the same street are willing to pay 2 rubles for installing each additional lantern. The neighbors' willingness to pay for this does not depend on the number of already installed lanterns. If the cost of setting up x lanterns is described by the function $C(x) = x^2$, find the Pareto-efficient amount of lanterns installed in the street.

Problem #6

There are three social groups in a large municipal district. The reverse demand functions of these groups for public TV broadcasting take the following forms: $P(t) = 150 - t$, $P(t) = 200 - 2t$, $P(t) = 250 - t$, where t stands for hours of broadcasting. Public TV broadcasting is a good supplied at a constant marginal cost equal 200 *groszy per* hour of broadcasting.

- Find the socially efficient amount of TV broadcasting hours.
- How many hours of broadcasting would a competitive private market supply?

Problem #1

An economy consists of two consumers having the following utility functions: $U_1(x_1, G) = 2x_1 + G$ and $U_2(x_2, G) = x_2 * G$ (good x is a private good and good G is a public good). Both goods are provided at the price level equal 1 (in other words, in the utility functions the public good is expressed in the units of the

private good). The total resources of the private good in the economy amount to 8000. Why is the allocation where $x_1 = 5000$, $x_2 = 1500$ and $G = 1500$ not Pareto-efficient?

Problem #2

Is it possible to determine the optimum level of street illumination by organizing a poll between several alternatives, where each inhabitant has one vote?

Problem #3

An economy consists of 2 consumers of a private good (the initial endowments of this good are for both consumers $\omega_1 = \omega_2 = 600$) and a public good, whose provision costs $c_1 = c_2 = 250$. The benefits from the latter good for the consumers are estimated as $v_1 = v_2 = 300$. Find the Groves-Clarke tax.

Problem #4

Consider an auction that requires providing bids in sealed envelopes (a so-called “sealed bid auction”). The auctioned good goes to the highest bidder but he/she pays the price equal to the second-highest bid. Provide your reasoning justifying that with such mechanism the bidders do not have incentives to bid differently than the value of the auctioned good. Justify as well that for an analogous mechanism but supplemented by the principle that the buyer pays the price equal to his/her bid, the bidders have incentives to decrease their bids (in relation to the value of the auctioned good).