Differentiating with respect to p_t and p_b, we see that the Kuhn-Tucker conditions for Row are

$$\begin{aligned} 2p_l &= \lambda + \mu_t \\ p_r &= \lambda + \mu_b. \end{aligned} \tag{15.1}$$

Since we already know the pure strategy solutions, we only consider the case where $p_t > 0$ and $p_b > 0$. The complementary slackness conditions then imply that $\mu_t = \mu_b = 0$. Using the fact that $p_t + p_b = 1$, we easily see that Row will find it optimal to play a mixed strategy when $p_l = 1/3$ and $p_r = 2/3$.

Following the same procedure for Column, we find that $p_t = 2/3$ and $p_b = 1/3$. The expected payoff to each player from this mixed strategy can be easily computed by plugging these numbers into the objective function. In this case the expected payoff is 2/3 to each player. Note that each player would prefer either of the pure strategy equilibria to the mixed strategy since the payoffs are higher for each player.

15.5 Interpretation of mixed strategies

It is sometimes difficult to give a behavioral interpretation to the idea of a mixed strategy. For some games, such as Matching Pennies, it is clear that mixed strategies are the only sensible equilibrium. But for other games of economic interest—e.g., a duopoly game—mixed strategies seem unrealistic.

In addition to this unrealistic nature of mixed strategies in some contexts, there is another difficulty on purely logical grounds. Consider again the example of the mixed strategy in the Battle of the Sexes. The mixed strategy equilibrium in this game has the property that if Row is playing his equilibrium mixed strategy, the expected payoff to Column from playing either of his pure strategies must be the same as the expected payoff from playing his equilibrium mixed strategy. The easiest way to see this is to look at the first-order conditions (15.1). Since $2p_l = p_r$, the expected payoff to playing top is the same as the expected payoff to playing bottom.

But this is no accident. It must always be the case that for any mixed strategy equilibrium, if one party believes that the other player will play the equilibrium mixed strategy, then he is indifferent as to whether he plays his equilibrium mixed strategy, or any pure strategy that is part of his mixed strategy. The logic is straightforward: if some pure strategy that is part of the equilibrium mixed strategy had a higher expected payoff than some other component of the equilibrium mixed strategy, then it would pay to increase the frequency with which one played the strategy with the higher expected payoff. But if all of the pure strategies that are played with positive probability in a mixed strategy have the same expected payoff, this must also be the expected payoff of the mixed strategy. And this in turn