EXAMPLE: Calculating a Nash equilibrium

The following game is known as the "Battle of the Sexes." The story behind the game goes something like this. Rhonda Row and Calvin Column are discussing whether to take microeconomics or macroeconomics this semester. Rhonda gets utility 2 and Calvin gets utility 1 if they both take micro; the payoffs are reversed if they both take macro. If they take different courses, they both get utility 0.

Let us calculate all the Nash equilibria of this game. First, we look for the Nash equilibria in pure strategies. This simply involves a systematic examination of the best responses to various strategy choices. Suppose that Column thinks that Row will play Top. Column gets 1 from playing Left and 0 from playing Right, so Left is Column's best response to Row playing Top. On the other hand, if Column plays Left, then it is easy to see that it is optimal for Row to play Top. This line of reasoning shows that (Top, Left) is a Nash equilibrium. A similar argument shows that (Bottom, Right) is a Nash equilibrium.

The Battle of the Sexes

Table 15.3

		Calvin	
		Left (micro)	Right (macro)
Rhonda	Top (micro)	2, 1	0, 0
	Bottom (macro)	0, 0	1, 2

We can also solve this game systematically by writing the maximization problem that each agent has to solve and examining the first-order conditions. Let (p_t, p_b) be the probabilities with which Row plays Top and Bottom, and define (p_l, p_r) in a similar manner. Then Row's problem is

$$\max_{(p_t,p_b)} \; p_t[p_l 2 + p_r 0] + p_b[p_l 0 + p_r 1]$$

$$\text{such that } p_t + p_b = 1$$

$$p_t \geq 0$$

$$p_b \geq 0.$$

Let λ, μ_t, and μ_b be the Kuhn-Tucker multipliers on the constraints, so that the Lagrangian takes the form:

$$\mathcal{L} = 2p_t p_l + p_b p_r - \lambda(p_t + p_b - 1) - \mu_t p_t - \mu_b p_b.$$