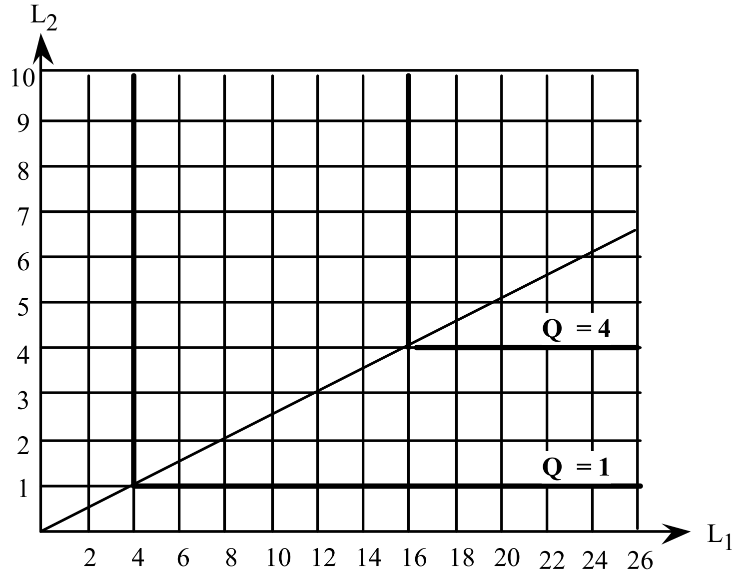
HOMEWORK I (solutions)

Problem 1

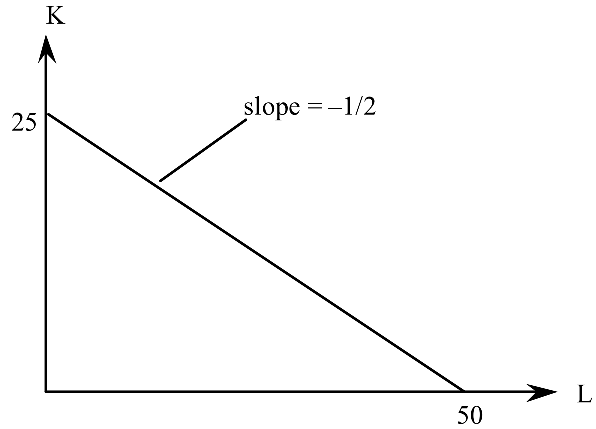
(a)



(b) $L_1 = 16$; $L_2 = 4$. When output was quadrupled, all inputs had to be increased by *the same* proportion. This production function exhibits *constant* returns to scale.
(c) With 8 pounds of lemons and one hour squeezing lemons, $Q = 1$. One more hour squeezing yields $\Delta Q = 1$, and so $MP_L = 1$.
(d) With 8 pounds of lemons and 2 hours of squeezing, $Q = 1$. She has no more lemons , and so the marginal product of labor is $MP_L = 0$.

Problem 2

(a) $K = 10$: $100 = 40 + 2L$, L = 30; $L = 25$: $100 = 4K + 50$, $K = 12.5$.
(b) $MRTS = 1/2$ and is constant; you can always trade $2L$ for $1K$; L and K are perfect substitutes.



(c) $Q = 40 + 2(31) = 102$; $MP_L = 2$ and doesn't change as more L is used.
(d) $Q = 4(13) + 50 = 102$; $MP_K = 2/0.5 = 4$ and it doesn't change.
(e) $Q = 4(20) + 2(20) = 120$; $Q = 4(40) + 2(40) = 240$; constant returns to scale, because Q doubled when both inputs were doubled.
(f) $Q = 2(28) + 4(11) = 100$; $2(-2) + 4(1) = 0$.

Problem 3

(a) The marginal product of L decreases for small increases in L holding K constant.
(b) The marginal product of K increases for small increases in K holding L constant.
(c) $MRTS_{LK} = MP_L/MP_K = (3L^{-1/2}K^{3/2})/9L^{1/2}K^{1/2} = (1/3)(K^{3/2}K^{-1/2})/(L^{1/2}L^{1/2}) = K/3L$.
(d) Yes.
(e) This technology exhibits *increasing* returns to scale. You can tell because the sum of the exponents in this Cobb-Douglas type production function is greater than unity.
(f) The law of diminishing marginal productivity.

Problem 4

Increase the use of factor 1 and decrease the use of factor 2.

Problem 5

No. If the inputs are perfect substitutes, the isoquants will be linear. However, to calculate the slope of the isoquant, and hence the MRTS, we need to know the rate at which one input may be substituted for the other. Without the marginal product of each input, we cannot calculate the MRTS.

Problem 6

The elasticity of substitution in case of the Leontief technology equals **0**.

Prove (for detailed prove please read the chapter on technology in advanced Varian):

The Leontief production function can be expressed by Constant Elasticity of Substitution (CES) production function (with $\rho = -\infty$):

$$f(x_1, x_2) = (a_1 x_1^\rho + a_2 x_2^\rho)^{1/\rho}$$

Let's set the parameters $a_1 = a_2 = 1$.

CES has a constant elasticity of substitution:

$$TRS = -\left(\frac{x_1}{x_2}\right)^{\rho-1}$$

so that

$$\frac{x_2}{x_1} = |TRS|^{\frac{1}{1-\rho}}$$

Taking logs, we see that

$$\ln\frac{x_2}{x_1} = \frac{1}{1-\rho}\ln|TRS|$$

Applying the definition of elasticity of substitution using the logarithmic derivative,

$$\sigma = \frac{d\ln x_2/x_1}{d\ln|TRS|} = \frac{1}{1-\rho} \xrightarrow{\rho\to-\infty} 0$$